PRACTICAL NECESSITY AS THE MOTHER OF STATISTICAL INNOVATION

CHRISTINE M. ANDERSON-COOK

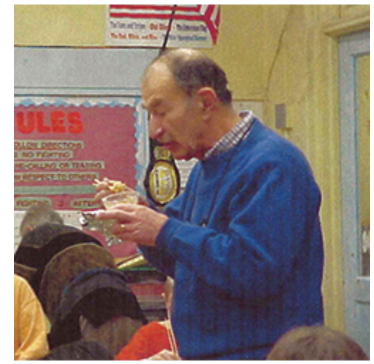
LOS ALAMOS, NM

candcook@gmail.com

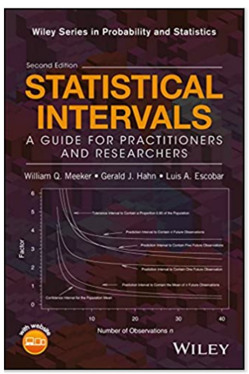
2021 Fall Technical Conference Gerald Hahn Talk

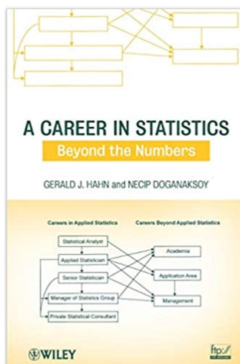
GERALD J. HAHN

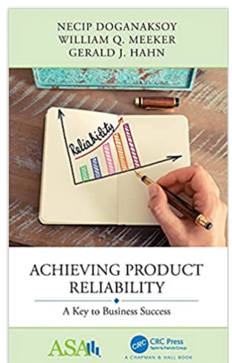
- 46 years at GE Global Research Center
- 26 years as Manager of the Applied Statistics Program
- First Chair of ASA Quality and Productivity Section

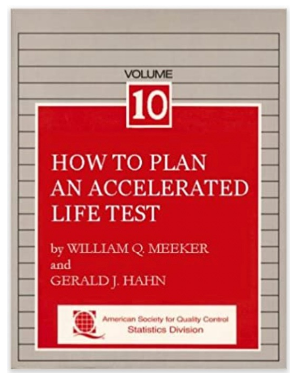


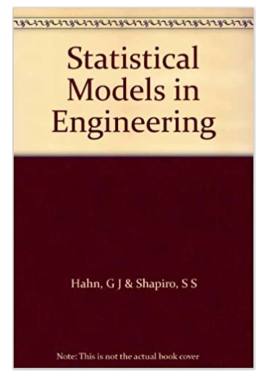
Role model for "leadership in developing, promoting, and successfully improving the quality and productivity of products and organizational performance using statistical concepts and methods"











Some of Gerry's hallmarks: Seeing what is important, articulating it clearly and leading to way to the right solution

REFLECTIONS ON MY CAREER

1994 – Ph.D. University of Waterloo (Advisor: C.F.J. Wu)
1994-1996 – Assistant Professor, Department of Statistics and Actuarial Science, Western University
1996-2004 – Assistant/Associate Professor, Department of Statistics, Virginia Tech
2002 – Visiting Professor at Arizona State University
2004-2021 – Research Scientist, Statistical Sciences Group, Los Alamos National Laboratory

Privileged to have my career span some exciting times in statistics:

Innovation in graphical capabilities

Renaissance of design of experiments

Dramatic increase in computational power for algorithmic searches

Dawn of "data science" and "big data"

Statistical Engineering (Hoerl and Snee, 2010) → ISEA (isea-change.org)

David Bellhouse (Western Ontario): "If you are lucky, you might have a handful of ideas in your career that really have a broader impact. The profession needs both explorers and settlers."

MY ROLE AS STATISTICIAN IN COLLABORATIONS

How my role has evolved throughout my career:

Early on:
Sales clerk approach

Customer comes in with a problem (want to collect some data)

Take their description and find the right product to solve the problem (create/find the right design to match needs)

Send them on their way, until they have another problem (often, help with the analysis)

In this talk, illustrations will focus on design of Experiments

MY ROLE AS STATISTICIAN IN COLLABORATIONS

Later:
Interior designer approach

Meet customer in their space (build context for data collection)

After understanding what is important to them, develop several possibilities (identify several options to compare)

Discuss what they like/dislike about options (deeper understanding of their priorities and context)

Iterate until they are satisfied with the final choice (build a deeper level of comfort for customer with design)

Key differences:
- Collaborative as a team member
- Deeper understanding of what is the right solution
- Multiple options discussed and compared
- Iterative – creating new designs is easy compared to executing the experiment
- Experimenter owns the final decision

SALES CLERK VS INTERIOR DESIGNER

Advantages of Designer Approach

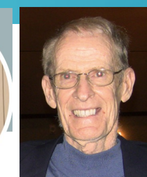
For organization:

- Class of problems that can be solved is much bigger
- Better definition of problem to be solved
- Better solutions
- Collaborators feel more ownership of statistical part of solution
- Collaborators gain insights into statistical expertise that they can use for other projects

For statistician:

- Contributions have potential for much larger impact
- Creates pull for involvement in subsequent projects
- Full team member status (not technical support)
- Greater professional fulfillment

HOW DID WE GET FROM THERE TO HERE?

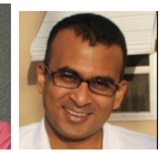
- Need #1: Richer set of tools to compare designs

Innovation: Fraction of Design Space (FDS) Plot

- Need #2: Ability to handle multi-faceted goals and protection from the unexpected


Pareto Fronts with flexible criteria

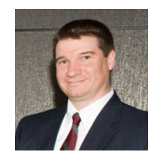
- Need #3: Ability to hedge risk from lack of knowledge with a "learn and leverage" approach

Sequential Design of Experiments (SDoE)

- Need #4: Improved structure for team discussions and decision-making

DMRCS, Collaboration & Graphical Tools

NEED #1 – TOOLS FOR COMPARING DESIGNS

- Need #1: Richer set of tools to compare designs

We make better decisions if we have a variety of choices:




We are much more engaged and discriminating about the fit of the solution to our problem if we have choices and can evaluate/compare their differences

Goal: Have a graphical and/or numerical summary that matches the priority of the study to compare the designs

NEED #1 – COMPARING DESIGNS (CONTINUED)

- Time travel back to 2000
- Goal: compare how well different designs predict throughout input space

Design Diagnostics

	Efficiency of Central Composite Design Relative to Box-Behnken
D-efficiency	1.220
G-efficiency	1.751
A-efficiency	1.066
I-efficiency	1.047
Additional Run Size	0

G-efficiency: Worst case prediction variance

I-efficiency: Average prediction variance

Problems:

- Too simplistic a summary for performance across a higher dimensional space
- We are going to use our model to predict at multiple input locations – not "average" or "worst"

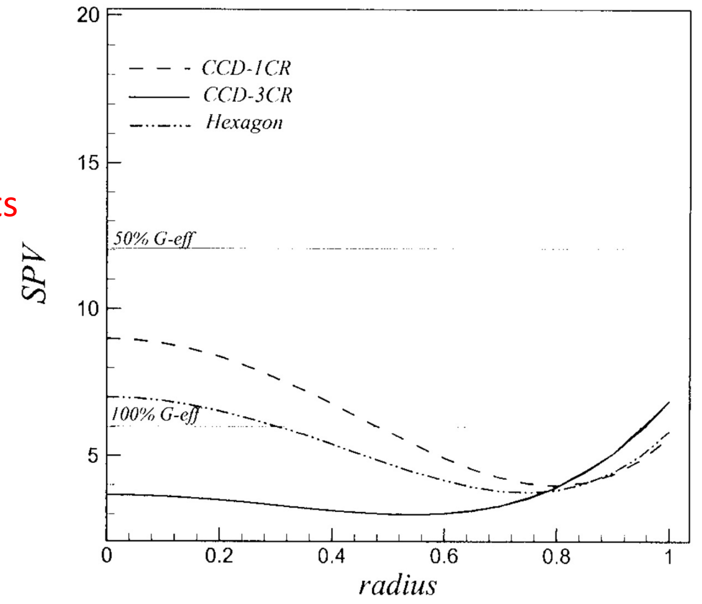
NEED #1 – COMPARING DESIGNS (CONTINUED)

- Time travel back to 2000
- Goal: compare how well different designs predict throughout input space

Problems:

- Relative emphasis changes with different # inputs
- Designed for spherical region
- Sometimes multiple (1-3) lines for each design
- Hard to know where to look

Variance-Dispersion Graph

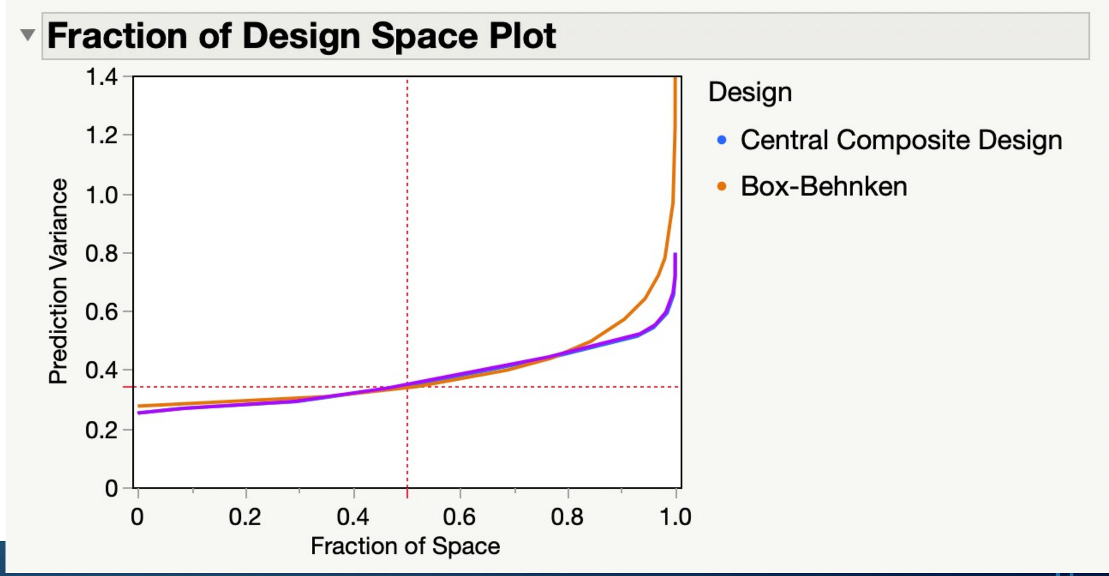


NEED #1 – COMPARING DESIGNS (CONTINUED)

- Single line for each design
- Same interpretation regardless of dimension, shape of region
- Comparison points easier to identify (best, worst, median)
- (Easy to generate)

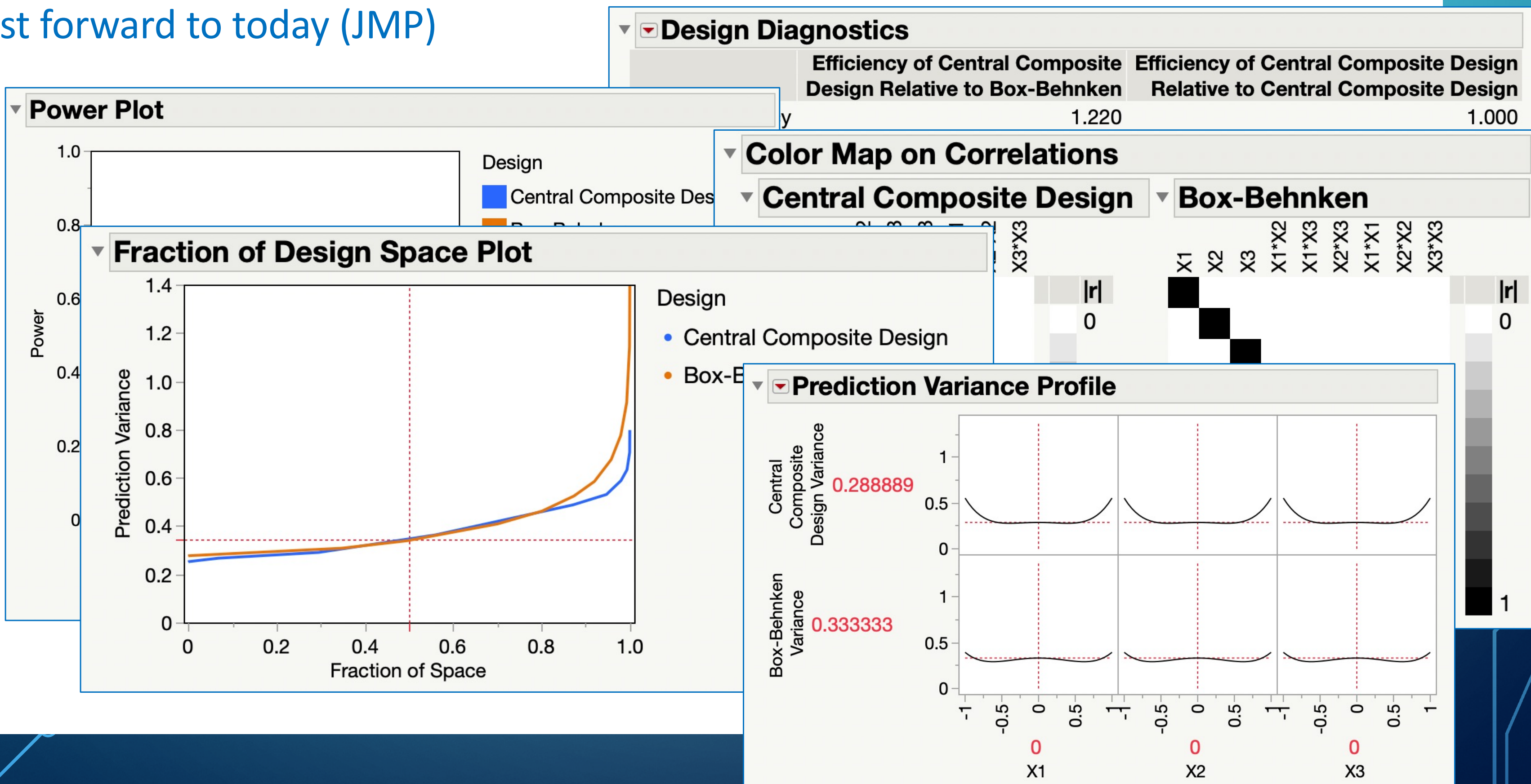
Versatile:

- Any shaped input region
- Compare different sizes
- Robustness to model choice
- Generalized linear model parameter choices



NEED #1 – COMPARING DESIGNS (CONTINUED)

Fast forward to today (JMP)



NEED #1 – COMPARING DESIGNS (CONTINUED)

Process for data collection planning with collaborators:

- Begin with discussion about priorities – what do success and failure look like?
- Creating designs is an iterative process – be responsive to new information, emphasize that creating a new design is easy
- Present multiple alternatives with their pros and cons
- Use graphical methods (like FDS plot, if appropriate) to demonstrate differences between choices
- Let experimenters make final choice of design between sensible alternatives

Greater comfort for experimenter about why this is the right design
Better chance for following implementation

HOW DID WE GET FROM THERE TO HERE?

- Need #1: Richer set of tools to compare designs

  Fraction of Design Space (FDS) Plot

- Need #2: Ability to handle multi-faceted goals and protection from the unexpected

  Pareto Fronts with flexible criteria

- Need #3: Ability to hedge risk from lack of knowledge with a "learn and leverage" approach

  Sequential Design of Experiments (SDoE)

- Need #4: Improved structure for team discussions and decision-making

  DMRCS, Collaboration & Graphical Tools

NEED 2: ABILITY TO HANDLE MULTI-FACETED GOALS AND PROTECTION FROM THE UNEXPECTED

There are many potential priorities to consider when designing an experiment

1. Result in good fit of the model to the data
2. Provide good model parameter estimates
3. Provide good prediction throughout the design space.
4. Provide an estimate of "pure" experimental error.

D-, A-optimal

I-, G-optimal

Good estimation and prediction for chosen model

5. Give sufficient information to allow for lack of fit test.
6. Provide a check on the homogeneous variance assumption.

Ability to test various aspects of the model

7. Be insensitive (robust) to the presence of outliers in the data.
8. Be robust to to errors in the control of design levels.

Protection if things go wrong

9. Allow models of increasing order to be constructed sequentially.
10. Allow for experiments to be done in blocks.

Flexibility to run and expand experiment

11. Be cost-effective.

Cost

Myers, Montgomery, Anderson-Cook RSM (2016) p. 370

NEED 2: ABILITY TO HANDLE MULTI-FACETED GOALS AND PROTECTION FROM THE UNEXPECTED

There are many potential priorities to consider when designing an experiment

1. Result in good fit of the model to the data
2. Provide good model parameter estimates
3. Provide good prediction throughout the design space.
4. Provide an estimate of "pure" experimental e

D-, A-optimal

I-, G-optimal

Good estimation and prediction for chosen model

5. Give sufficient information to allow f
6. Provide a check on the hom
assumption.

Ab ity to test various cts of the model

7. Be insensitiv outliers i
the d
8. P ontrol of

easing

ty.

10 of experim

11. Be cost-effective.

on if things go wrong

Flexibility to run and expand experiment

Cost

"Optimal" is conditional on (1) a particular model and (2) the experiment being executed the way it was planned

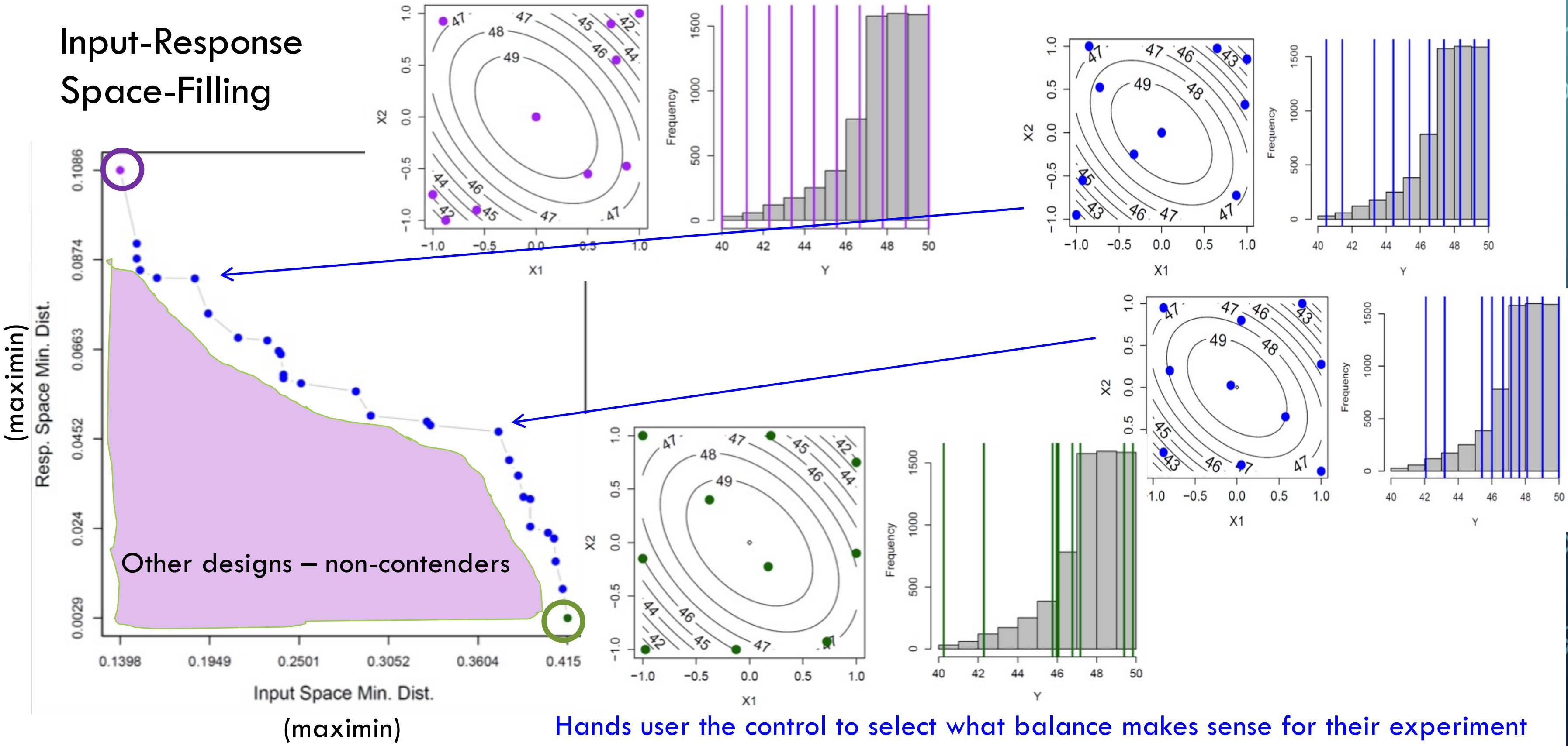
We are often seeking to squeeze more and more out of each experiment – one criterion might be too narrow

Myers, Montgomery, Anderson-Cook RSM (2016) p. 370

NEED 2 (CONTINUED)

PARETO FRONT BASICS

Input-Response Space-Filling



Hands user the control to select what balance makes sense for their experiment

NEED 2 (CONTINUED)

Some Examples in the Literature

1. **Screening Experiment**:
   - D-optimality [maximize |X'X|]
   - Good estimation of pure error [maximize df_{PE}]
   - Good estimation of lack of fit [maximize tr(R'R) /(m-p)]
2. **Robust Parameter Design Experiment**
   - Good estimation of terms affecting the mean [max D_s-mean]
   - Good estimation of terms affecting the variance [max D_s-variance]
   - Size of experiment [min N]
3. **Split Plot Design**
   - Good estimation of terms when WP to SP variance ratio is unknown [max D(0.1), max D(10)]
   - Size of experiment [min N]
   - Number of Whole Plots [min #WP]
4. **Sequential Experimentation for Estimating System Reliability**
   - Good prediction of
     - system reliability [min uncertainty interval]
     - mechanical sub-system reliability [min uncertainty interval]
     - electrical sub-system reliability [min uncertainty interval]
5. **Space-Filling Design**
   - Good spacing in input space [maximin in X]
   - Good spacing in predicted response range. [maximin in Y]

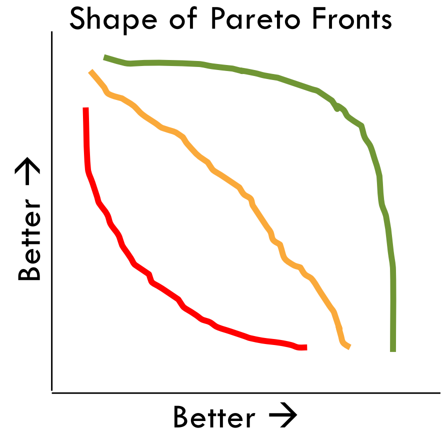
NEED 2: ABILITY TO HANDLE MULTI-FACETED GOALS AND PROTECTION FROM THE UNEXPECTED SUMMARY

Experience has taught me that:

1. Getting the model right before you have collected the data is not common or easy.
2. Different stakeholders have different priorities for the experiment. Don't require everyone to come to consensus on one goal.
3. A close-to-optimal design that can survive some surprises is better than a non-robust optimal design.

The Pareto front approach allows for:

- Multiple user-selected priorities to be considered
- Multiple competitive designs to be created and compared
- Graphical tools to facilitate discussion about alternatives
- Understanding of how severe the trade-offs are for different priorities
- Intentional protection from things not going as planned (imperfect knowledge and/or problems running experiment)



HOW DID WE GET FROM THERE TO HERE?

- Need #1: Richer set of tools to compare designs

  Fraction of Design Space (FDS) Plot

- Need #2: Ability to handle multi-faceted goals and protection from the unexpected

  Pareto Fronts with flexible criteria

- Need #3: Ability to hedge risk from lack of knowledge with a "learn and leverage" approach

  Sequential DoE and NUSF

- Need #4: Improved structure for team discussions and decision-making

  DMRCS, Collaboration & Graphical Tools

NEED 3: HEDGE RISK WITH A "LEARN AND LEVERAGE" APPROACH

Some of my favorite solutions for incorporating knowledge into experiments:

1. Sequential Design of Experiments

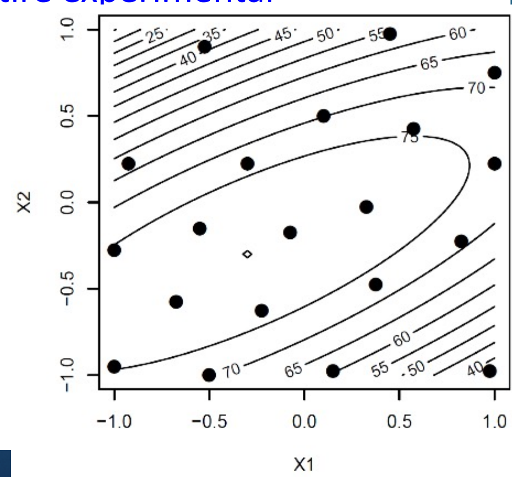
If you are going on a long trip, would you rather:

1. Commit to exactly the route, street-by-street at the start of your journey
2. Plan the first part of your route, and then each evening adapt the next step of the journey using updated information

Same logic applies to collecting data – why commit the entire experimental budget at the beginning? Why not learn as we go?

2. Non-Uniform Space-Filling Designs

Space-filling designs that provide the experimenter with control to change the density of points to match their level of interest in a region



SEQUENTIAL DESIGN OF EXPERIMENTS

1. If it is possible to collect, measure and analyze the data in a **timely** fashion.
   – Makes implementation possible.

2. If the process is **stable and consistent** over time.
   – Makes combining data from various stages straightforward
3. If there is **uncertainty about the input region** of interest.
   – Avoid wasting resources in undesirable regions
4. If the experiment has **several objectives**.
   – Easier to tackle objectives individually and leverage what is already known.
5. If **little is known** about the process.
   – Learn as you go.
6. If logistics impose **constraints on the size of the experiments** that could be run at any time window.
   –Natural if blocking inherently imposed

SEQUENTIAL DESIGN OF EXPERIMENTS

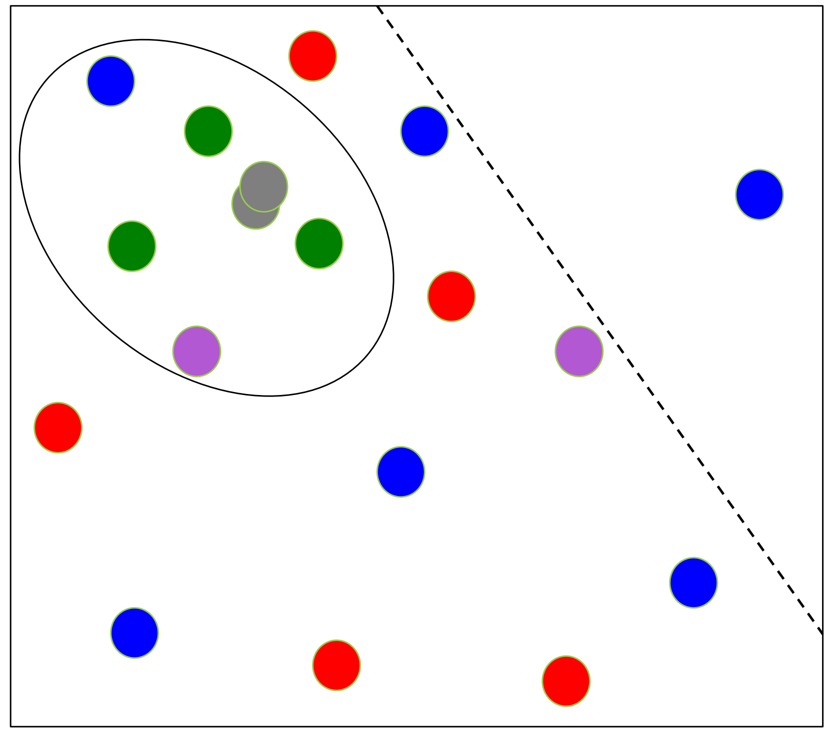
Pilot study — Can we get quality data and measure what we need?

Exploration — Understand basic relationship between inputs and response

Model Building/ Refinement — Verify model adequately characterizes relationship with acceptable level of uncertainty

Optimization — Focus on portion of region with best values of the response

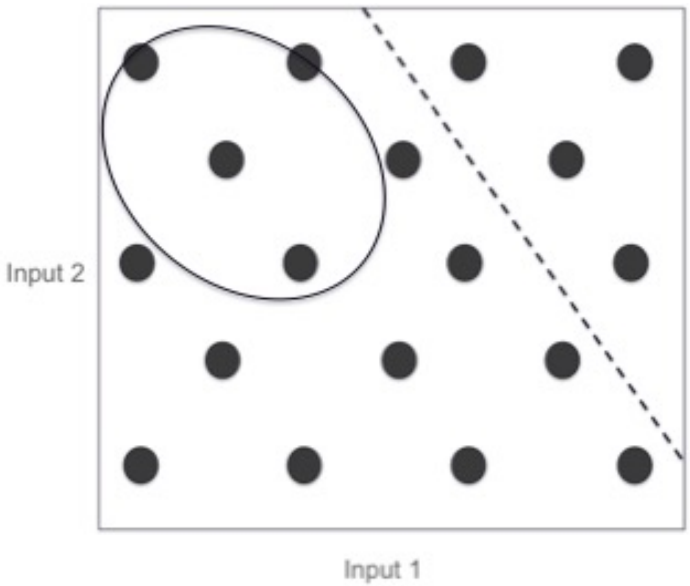
Confirmation — Verify results for production or operational use

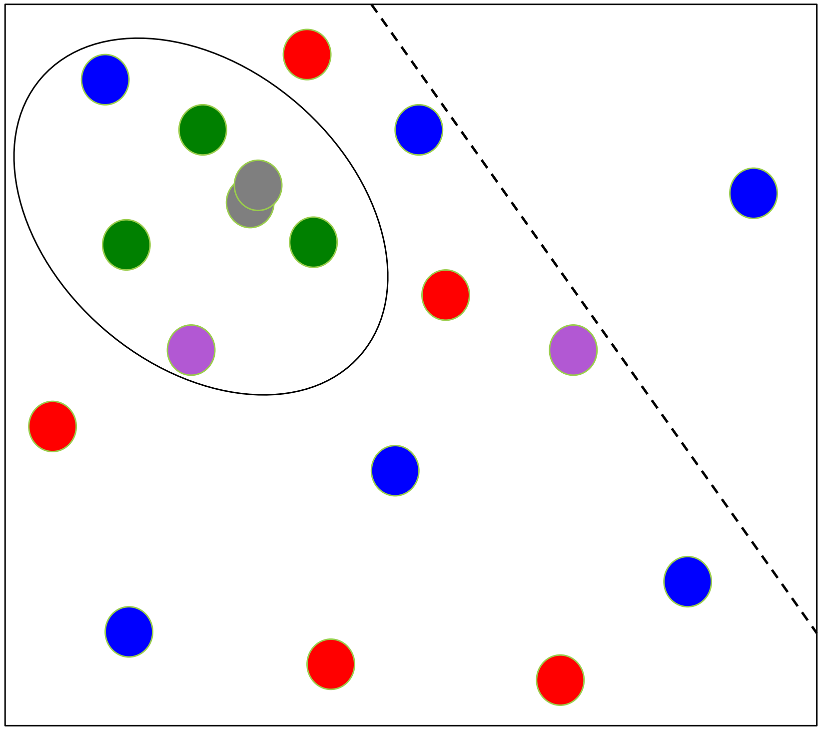


Conceptual Space-Filling Example – 2 factors 18 run budget

SEQUENTIAL DESIGN OF EXPERIMENTS

One-Shot Experiment





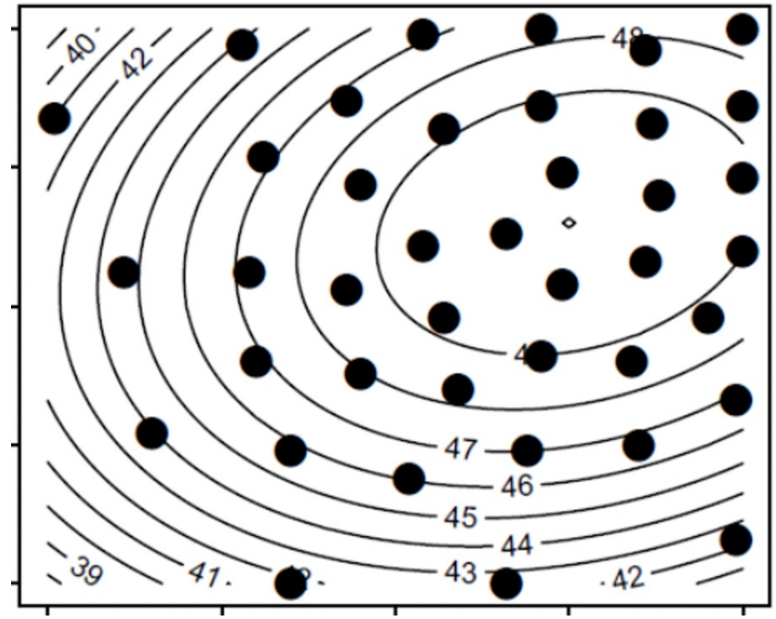
Conceptual Example – 2 factors 18 run budget

NON-UNIFORM SPACE-FILLING (NUSF) DESIGNS

Key to NUSF: define weights for each location in the input space that reflect experimenter interest

Assign higher weights to regions where:

- Response values are better (optimize)
- More interesting features (exploration)
- Response changing more quickly (exploration)
- Prediction has larger uncertainty (model refinement)
- Discrepancy is larger between computer model and observed data (calibration)



NON-UNIFORM SPACE-FILLING (NUSF) DESIGNS

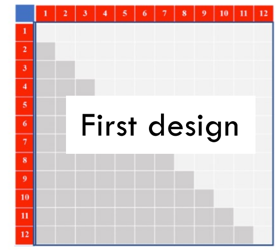
Idea:

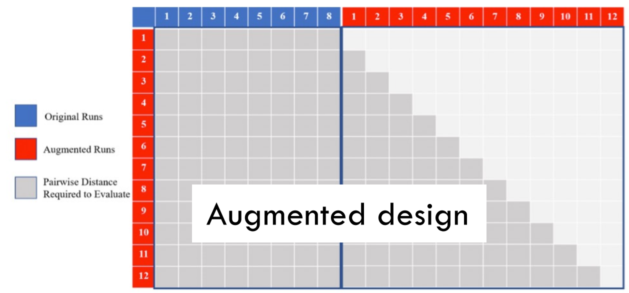
- User specifies a weight function for all points in their candidate set (larger weights for regions with higher density of points)
- Distance function for *maximin* criterion is adjusted from

$$d(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sqrt{\sum_{j=1}^{p} \left(x_{1j} - x_{2j}\right)^2} \quad \text{to}$$

$$d^w(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sqrt{w(\boldsymbol{x}_1) w(\boldsymbol{x}_2) d(\boldsymbol{x}_1, \boldsymbol{x}_2)^2}$$

Weights associated with each candidate point.





To tune the degree of non-uniformity: scale raw weights from [Min, Max] → [1, MWR]

If MWR = 1 ←→ uniform

as MWR increases, degree of non-uniformity increases

NON-UNIFORM SPACE-FILLING (NUSF) DESIGNS

Idea:

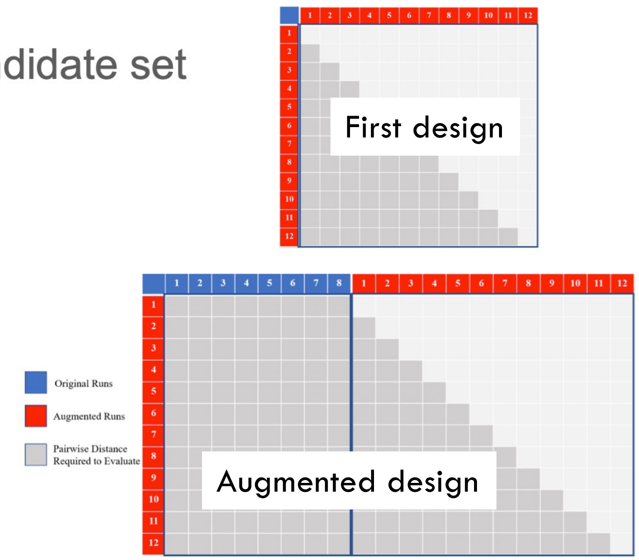
- User specifies a weight function for all points in their candidate set (larger weights for regions with higher density of points)
- Distance function for *maximin* criterion is adjusted from

$$d(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sqrt{\textstyle\sum_{j=1}^{p} \left(x_{1j} - x_{2j}\right)^2}$$

to

$$d^w(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sqrt{w(\boldsymbol{x}_1) w(\boldsymbol{x}_2) d(\boldsymbol{x}_1, \boldsymbol{x}_2)^2}$$

Weights associated with each candidate point.



To tune the degree of non-uniformity: scale raw weights from [Min, Max] → [1, MWR]

If MWR = 1 ←→ uniform as MWR increases, degree of non-uniformity increases

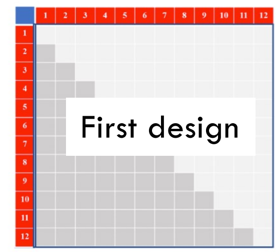
NON-UNIFORM SPACE-FILLING (NUSF) DESIGNS

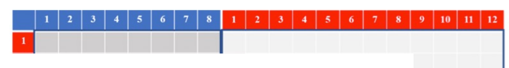
Idea:

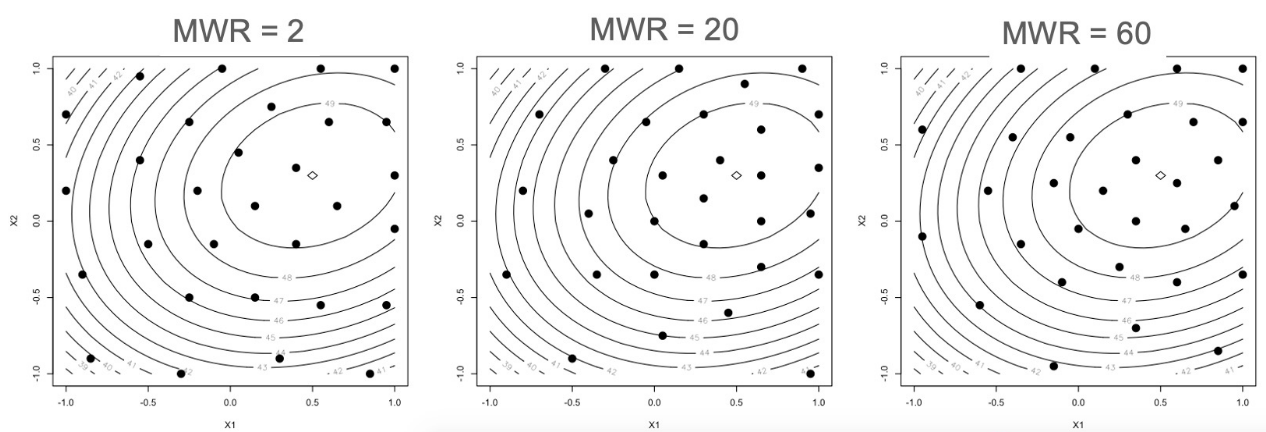
- User specifies a weight function for all points in their candidate set (larger weights for regions with higher density of points)
- Distance function for *maximin* criterion is adjusted from

$$d(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sqrt{\sum_{j=1}^{p} \left(x_{1j} - x_{2j}\right)^2}$$ to

$$d^w(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sqrt{}$$







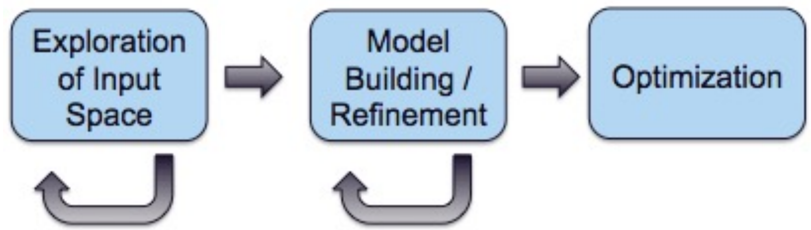
Maximum Weight Ratio: Dial to tune degree of non-uniformity

Weights associated

To tune the degı R]

If MWR = 1 ←→ ies

BUILDING A SEQUENCE OF EXPERIMENTS



Case 1: Exploration	USF Augmented USF to fill in (same or refined region)		
Case 2: Optimization	USF	Augm NUSF To reduce uncertainty	Augm NUSF To target optimum
Case 3: Prediction throughout	USF	Augm IRSF To balance spacing of inputs and responses	

HOW DID WE GET FROM THERE TO HERE?

- Need #1: Richer set of tools to compare designs

  Fraction of Design Space (FDS) Plot

- Need #2: Ability to handle multi-faceted goals and protection from the unexpected

  Pareto Fronts with flexible criteria

- Need #3: Ability to hedge risk from lack of knowledge with a "learn and leverage" approach

  Sequential DoE and NUSF

- Need #4: Improved structure for team discussions and decision-making

  DMRCS, Collaboration & Graphical Tools

NEED 4: IMPROVED STRUCTURE FOR TEAM DISCUSSIONS AND DECISION-MAKING

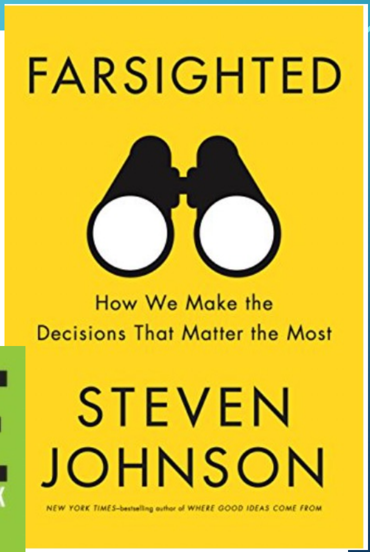
We are only effective as statisticians when our voices and message are heard.

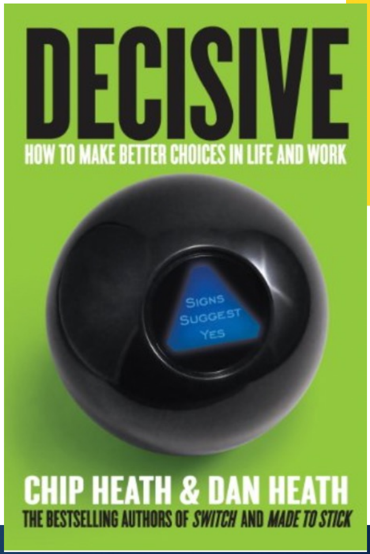
Some of ideas:

- DMAIC: Define-Measure-Analyze-Improve-Control for Problem-Solving
- DMRCS: Define-Measure-Reduce-Combine-Select for Decision-Making
- Facilitating Discussion and Decision-Making
- Collaboration Strategies
- STATISTICAL ENGINEERING

COMMON PROBLEMS WITH DECISION-MAKING

1. Narrow framing: limiting choices and options
2. Confirmation bias: seeing and evaluating data based on current leading choice
3. Thinking too short-term: not including longer term consequences into decision
4. Overconfidence: not building in sufficient uncertainty about future
5. Team-dynamics: personalities, process, competing priorities





NEED 4: IMPROVED STRUCTURE FOR TEAM DISCUSSIONS AND DECISION-MAKING

Example: Choosing a Design

Define
- What is important for this experiment?
- What designs are available to choose between?

Measure
- What is the right criterion to measure the important attribute?
- High quality data to assess designs for identified criteria?
- Gather relevant information

Reduce
- Are some aspects secondary in importance? Remove or defer?
- Can some designs be eliminated as implausible or non-contenders?

Combine
- How can I evaluate trade-offs between criteria (that are probably on different scales)?
- How much do I value the different criteria?

Select
- Which solution is best given my priorities?
- How does this solution compare to other options?
- Can I defend my choice?

NEED 4: IMPROVED STRUCTURE FOR TEAM DISCUSSIONS AND DECISION-MAKING

Anderson-Cook, C.M., Lu, L. (2018) "Graphics to Facilitate Informative Discussion and Team Decision-Making" **Applied Stochastic Models in Business and Industry** (with discussion and rejoinder)

	General
G1	Use a process to find common ground
G2	Clarify the difference between "right and wrong" and "choosing differently based on priorities"
	Define
D1	Define and use an appropriate summary that directly connects to the decision
D2	Build the decision space to include diverse alternatives
D3	Push the boundaries on assumptions
D4	Incorporate cost into the comparisons of alternatives
	Measure
M1	For realistic decision making, it is important to provide uncertainty quantification to inform the decision makers about the uncertainty and potential risks associated with a specific decision
M2	Devise analysis summaries that reduce the uncertainty can improve the decision-making process

NEED 4: IMPROVED STRUCTURE FOR TEAM DISCUSSIONS AND DECISION-MAKING

	Reduce
R1	It is helpful to take strategic steps to reduce the number of choices on which to do a detailed comparison to a manageable number
	Combine
C1	Allow common visualization and discussion about results, instead of keeping subjective elements of the decision unshared
C2	When comparing the smaller set of contenders, choose plots that highlight the impact of subjective choices to facilitate discussion
C3	Think globally and locally
C4	Consider dynamic graphics when dimensionality of problem suggests it
	Select
S1	Create or use a graphical summary that appropriately captures the needed level of detail
S2	Include graphics to formalize conclusions

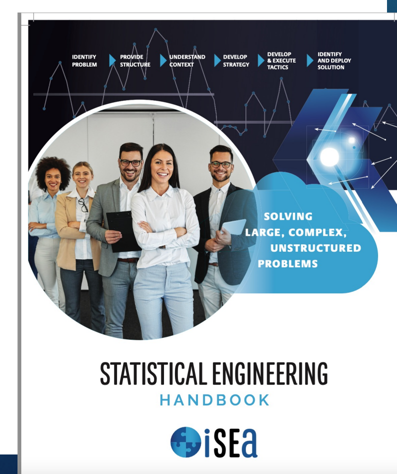
NEED 4: IMPROVED STRUCTURE FOR TEAM DISCUSSIONS AND DECISION-MAKING

Collaboration Strategies

- Anderson-Cook, C.M., Lu, L., Parker, P.A. (2019) "Effective Interdisciplinary Collaboration Between Statisticians and Other Subject Matter Experts" **Quality Engineering** (with discussion and rejoinder)
- 2020 FTC talk: https://www.youtube.com/watch?v=9PUqc2puBag

STATISTICAL ENGINEERING

- International Statistical Engineering Association (free membership isea-change.org)
- **ISEA Workshop: Nov 18-19**
- 2012 Quality Engineering Special Issue (https://www.tandfonline.com/toc/lqen20/24/2?nav=tocList)



CONCLUSIONS

We needed several key elements:

- Ability to generate multiple designs that allow for comparisons
- Ability to have the right tools/graphics to make specific and meaningful comparisons
- Ability to collect data, learn from it, pivot using what we learn, collect the next set of data
- Develop "human skills*", confidence and strategies to lead and/or be full team members on collaborative teams

We are in exciting times for statistics – many new areas emerging, where we can make important contributions.

Consider new ideas to help improve problem-solving and decision-making.

Let the needs of collaborators, practitioners guide our innovation.

*Simon Sinek

REFERENCES FOR FRACTION OF DESIGN SPACE PLOTS

1. Zahran, A., Anderson-Cook, C.M. and Myers, R.H. (2003) "Fraction of Design Space to Assess the Prediction Capability of Response Surface Designs" **Journal of Quality Technology** 35 377-386.
2. Goldfarb, H.B., Anderson-Cook, C.M., Borror, C.M., Montgomery, D.C. (2004) "Fraction of Design Space to Assess the Prediction Capability of Mixture and Mixture-Process Designs" **Journal of Quality Technology** 36 169-179.
3. Ozol-Godfrey, A., Anderson-Cook, C.M., Montgomery, D.C. (2005) "Fraction of Design Space Plots for Examining Model Robustness" **Journal of Quality Technology** 37 223-235.
4. Liang, L., Anderson-Cook, C.M., Robinson, T.J. (2006) "Fraction of Design Space Plots for Split-Plot Designs" **Quality and Reliability Engineering International** 22 275-289.
5. Ozol-Godfrey, A., Anderson-Cook C. M. (2007) "Fraction of Design Space Plots for Examining Mixture Design Robustness to Measurement Errors" **Journal of Statistics and Applications** 1 171-183.
6. Ozol-Godfrey, Anderson-Cook, C.M., Robinson, T.J. (2008) "Fraction of Design Space Plots for Generalized Linear Models" **Journal of Statistical Planning and Inference** 138 203-219.
7. Jang, D.-H., Anderson-Cook, C.M. (2011) "Fraction of Design Space Plots for Evaluating Ridge Estimators in Mixture Experiments" **Quality and Reliability Engineering International** 27 27-34.

REFERENCES FOR PARETO FRONT DESIGNS

1. Lu, L., Anderson-Cook, C.M., Robinson, T.J. (2011) "Optimization of Designed Experiments Based on Multiple Criteria Utilizing a Pareto Frontier" **Technometrics** 53 353-365.
2. Lu, L., Anderson-Cook, C.M., Robinson, T.J. (2012) "A Case Study to Demonstrate Pareto Frontiers for Selecting a Best Response Surface Design with Simultaneously Optimizing Multiple Criteria" **Applied Stochastic Models in Business and Industry** 28 206-221.
3. Lu L., Anderson-Cook, C.M. (2012) "Rethinking the Optimal Response Surface Design for a First-Order Model with Two-Factor Interactions, when Protecting against Curvature" **Quality Engineering** 24 404-422.
4. Lu, L., Chapman, J., Anderson-Cook, C.M. (2013) "A Case Study on Selecting A Best Allocation of New Data for Improving the Estimation Precision of System and Sub-System Reliability using Pareto Fronts" **Technometrics** 55 473-487.
5. Lu, L., Anderson-Cook, C.M. (2013) "Adapting the Hypervolume Quality Indicator to Quantify Trade-Offs and Search Efficiency for Multiple Criteria Decision-Making Using Pareto Fronts" **Quality and Reliability Engineering International** 29 1117-1133
6. Lu, L., Anderson-Cook, C.M. (2014) "Balancing Multiple Criteria Incorporating Cost using Pareto Front Optimization for Split-Plot Designed Experiments" **Quality and Reliability Engineering International** 30 37-55
7. Lu, L., Anderson-Cook, C.M., Lin, D. (2014) "Optimal Designed Experiments Using a Pareto Front Search for Focused Preference of Multiple Objectives" **Computational Statistics and Data Analysis** 71 1178-1192
8. Chapman, J., Lu, L., Anderson-Cook, C.M. (2014) "Process Optimization for Multiple Responses Utilizing the Pareto Front Approach" **Quality Engineering** 26 253–268.
9. Chapman, J.L., Lu, L., Anderson-Cook, C.M. (2014) "Incorporating Response Variability and Estimation Uncertainty into Pareto Front Optimization" **Computers & Industrial Engineering** 76 253-267

REFERENCES FOR SEQUENTIAL DESIGN OF EXPERIMENTS

1. Lu, L., Anderson-Cook, C.M. (2021) "Strategies for Sequential Design of Experiments and Augmentation" **Quality and Reliability Engineering International** 37(5) 1740-1757.
2. Lu, L., Anderson-Cook, C.M., Ahmed, T. (2020) "Non-Uniform Space-Filling Designs" **Journal of Quality Technology** 53(3) 309-330.
3. Lu, L., Anderson-Cook, C.M. (2021) "Input-Response Space-Filling Designs" **Quality and Reliability Engineering International** http://doi.org/10.1002/qre.2931
4. Anderson-Cook, C.M. Lu, L. (2020) "Why Not Sequential DoEs?" **Quality Progress** 52(11) 44-47.
5. Anderson-Cook, C.M., Graves, T.G., Hamada, M. (2008) "Resource Allocation: Sequential Design for Analyses Involving Several Types of Data" IEEE Xplore Refereed Proceedings of IEEM Singapore

REFERENCES FOR STATISTICAL ENGINEERING & COLLABORATION

1. Anderson-Cook, C.M., Lu, L. (2015) "Much-Needed Structure: A New 5-Step Decision-Making Process Helps You Evaluate, Balance Competing Objectives" **Quality Progress** 48(10) 42-50.
2. Anderson-Cook, C.M., Lu, L., Parker, P.A. (2019) "Effective Interdisciplinary Collaboration Between Statisticians and Other Subject Matter Experts" **Quality Engineering** (with discussion and rejoinder) 31(1) 164-204.
3. Anderson-Cook, C.M., Lu, L. (2018) "Graphics to Facilitate Informative Discussion and Team Decision-Making" **Applied Stochastic Models in Business and Industry** (with discussion and rejoinder) 34 (6) 963-991.
4. Anderson-Cook, C.M., Lu, L., Clark, G., DeHart, S.P., Hoerl, R., Jones, B., MacKay, R.J., Montgomery, D.C., Parker, P.A., Simpson, J., Snee, R., Steiner, S., Van Mullekom, J., Vining, G.G., Wilson, A.G. (2012) "Statistical Engineering – Forming the Foundations" **Quality Engineering** 24 110-132.
5. Anderson-Cook, C.M., Lu, L., Clark, G., DeHart, S.P., Hoerl, R., Jones, B., MacKay, R.J., Montgomery, D.C., Parker, P.A., Simpson, J., Snee, R., Steiner, S., Van Mullekom, J., Vining, G.G., Wilson, A.G. (2012) "Statistical Engineering – Roles for Statisticians and the Path Forward" **Quality Engineering** 24 133-152.