Geometric Shape Deviation Modeling Across Different Processes and Shapes in Additive Manufacturing Systems

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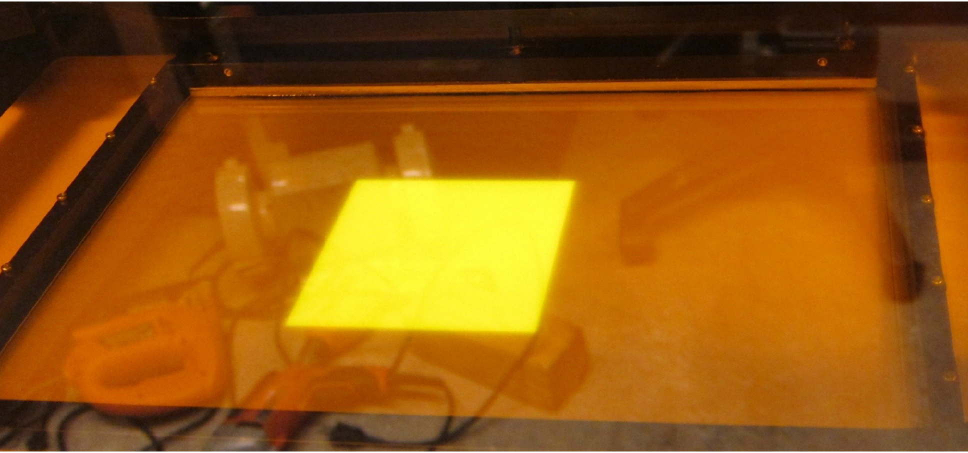
Outline

1. Background: Challenges in additive manufacturing (AM) systems
2. Objective: Shape deviation modeling across different processes and shapes in AM systems
3. Mean effect equivalence (EE) framework and methodology
4. Bayesian learning of deviation features for different shapes
5. Combining mean EE with deviation features for comprehensive deviation modeling in AM systems
6. Concluding remarks and discussion

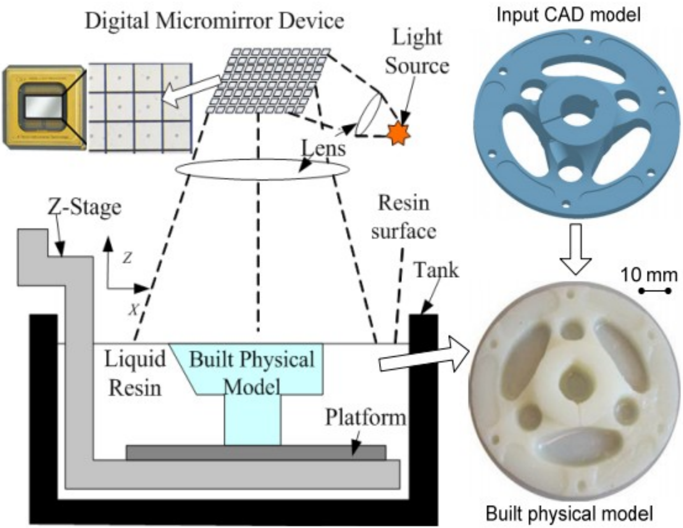
Additive Manufacturing: A Disruptive Technology

Our AM Technology: Stereolithography

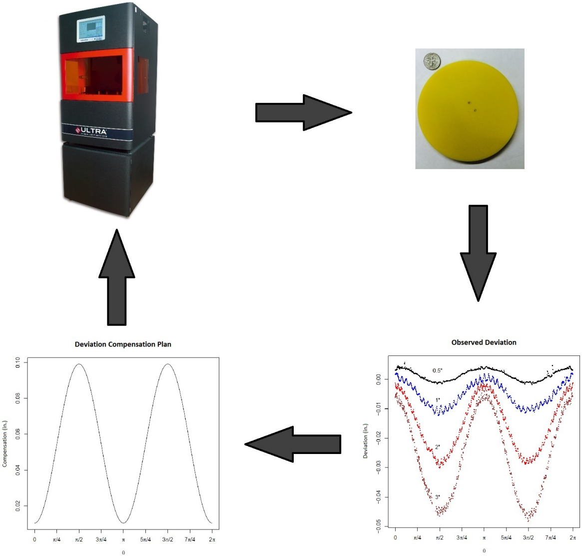
Stereolithography in Action



Review of Stereolithography and Shape Deviation

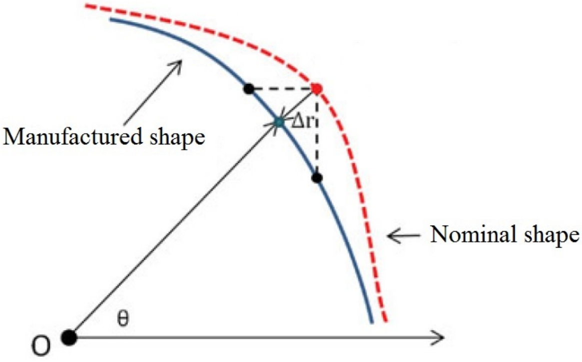


Big Picture of Accuracy Control via Deviation Models



Huang Q., Zhang J., Sabbaghi A., Dasgupta T. (2015). Optimal offline compensation of shape shrinkage for 3D printing processes. *IIE Transactions on Quality and Reliability Engineering*, 47(5): 431–444.

Deviation Representation (Huang et al., 2015)

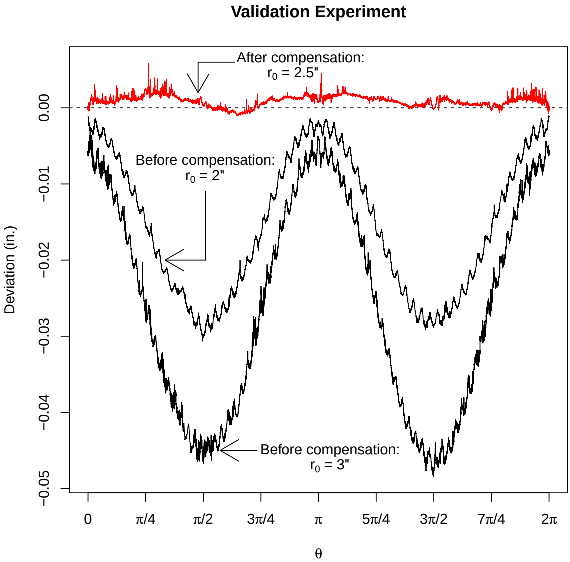


Nominal radius function for shape i: $r_i^{\text{nom}} : [0, 2\pi] \to \mathbb{R}_{>0}$.

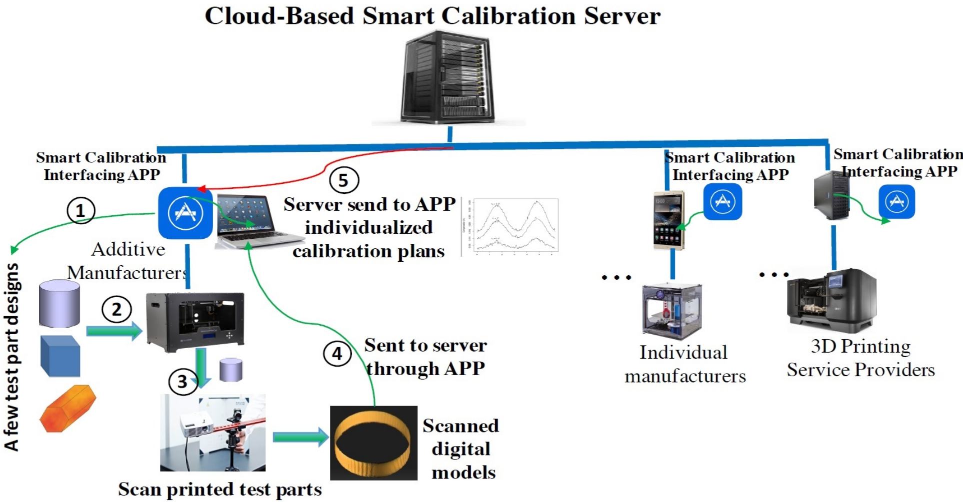
Deviation for point θ on shape i under compensation x_1:

$$\Delta_i(\theta, x_1) = r_i^{\text{obs}}(\theta, x_1) - r_i^{\text{nom}}(\theta).$$

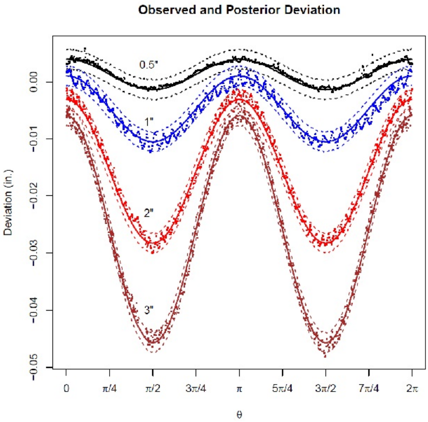
Validation Experiment (Huang et al., 2015: p. 439)



Accuracy Control for Cyber-Physical AM Systems

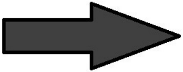


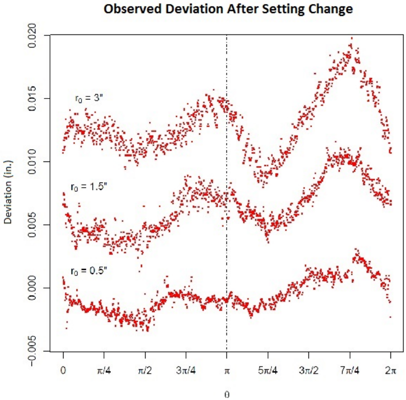
Challenge: Heterogeneous Process Conditions



$$\underbrace{X_1 = x_1}_{\text{Observed factor}} \;,\; \underbrace{X_2 = c_2}_{\text{Lurking variable}} \longrightarrow \underbrace{p(y \mid x_1)}_{\text{Deviation model}}$$

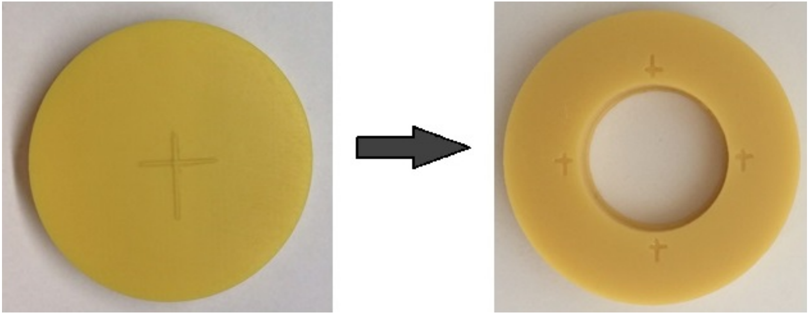
Challenge: Heterogeneous Process Conditions



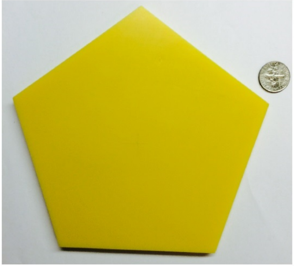


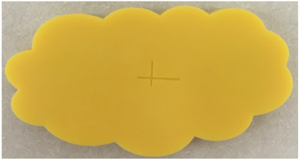
$$\underbrace{X_1 = \color{red}{x_1}}_{\text{Observed factor}} \quad , \quad \underbrace{X_2 = \color{red}{x_2}}_{\text{Lurking variable}} \longrightarrow \underbrace{?}_{\text{Deviation model}}$$

Challenge: Heterogeneous Process Conditions

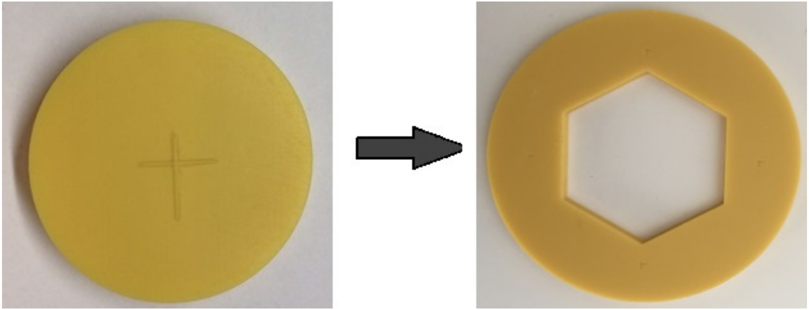


Challenge: Huge Varieties and Shape Complexities





Challenge: Different Processes and Shapes



Overview of Challenges and Objectives

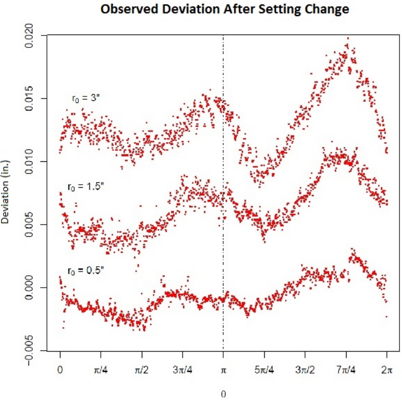
Geometric shape deviation models constitute an important component in dimensional accuracy control for additive manufacturing (AM) systems.

Model building in AM systems is made difficult by their

- vast spectrum of distinct process conditions,
- wide varieties of complex shapes, and
- low-volume production (one-of-a-kind manufacturing).

The paradigm shift introduced by AM systems motivates our development of new Bayesian and machine learning methodologies for comprehensive deviation modeling.

Model Transfer via Mean Effect Equivalence in AM



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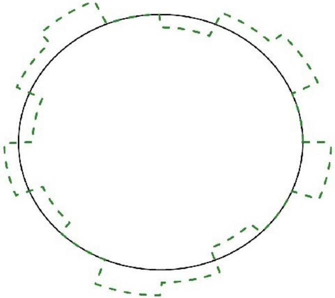
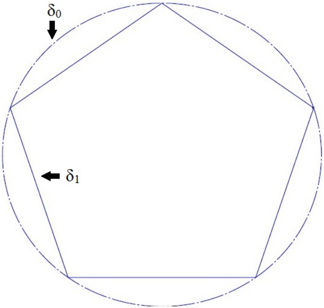


$$\underbrace{X_1 = x_1}_{\text{Observed factor}}, \underbrace{X_2 = x_2}_{\text{Lurking variable}} \equiv \underbrace{X_1 = T}_{\text{Total equivalent amount}}, \underbrace{X_2 = c_2}_{\text{Base setting}}$$

Sabbaghi A., Huang Q. (2018). Model transfer across additive manufacturing processes via mean effect equivalence of lurking variables. *Annals of Applied Statistics*. (in press).

Bayesian Learning From Small Samples of Distinct Shapes



Known deviation feature for previous shape: δ_0.

New feature unique to new shape that is to be learned: δ_1.

$$\text{Deviation for new shape} = \delta_0 + \delta_1$$

Sabbaghi A., Huang Q., Dasgupta T. (2018). Bayesian model building from small samples of disparate data for capturing in-plane deviation in additive manufacturing. *Technometrics* (in press).

Strategy: Effect Equivalence and Deviation Features

Three methods underlie our strategy for comprehensive deviation modeling in AM systems.

1. Functional deviation representation (Huang et al., 2015)
2. Mean effect equivalence (Wang et al., 2005; Sabbaghi & Huang, 2018)
3. Bayesian learning of modular deviation features (Huang et al., 2014; Sabbaghi et al. 2018)

We illustrate our strategy for in-plane deviation modeling of cylinders, polygons, and cavities under different stereolithography processes.

Sabbaghi A., Huang Q. (2016). Predictive model building across difference process conditions and shapes in 3D printing. *Proceedings of the Twelfth Annual IEEE International Conference on Automation Science and Engineering*, August 2016.

Definition of Mean Effect Equivalence

Let F_k denote the factors for an AM process, $\mathcal{X}_k$ their set of levels, and $\mathbf{z}$ the covariate vector for a point on shape i.

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Let $p(y|\mathbf{z}, x_1, x_2, \psi)$, $p_1(y|\mathbf{z}, x_1, \psi_1)$, and $p_2(y|\mathbf{z}, x_2, \psi_2)$

denote the probability density functions for

$\Delta_i(\theta, x_1, x_2)$, $\Delta_i(\theta, x_1, c_2)$, and $\Delta_i(\theta, c_1, x_2)$.

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Definition

Factors F_1 and F_2 are equivalent with respect to the mean if for any $c_1 \in \mathcal{X}_1$ and $c_2 \in \mathcal{X}_2$, functions $T_{1\to 2} : \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_2$ and $T_{2\to 1} : \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{X}_1$ exist such that for all $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$:

$$\int_{-\infty}^{\infty} yp(y|\mathbf{z}, x_1, x_2, \psi)dy = \int_{-\infty}^{\infty} yp_1(y|\mathbf{z}, T_{2\to 1}(x_1, x_2), \psi_1)dy,$$

$$\int_{-\infty}^{\infty} yp(y|\mathbf{z}, x_1, x_2, \psi)dy = \int_{-\infty}^{\infty} yp_2(y|\mathbf{z}, T_{1\to 2}(x_1, x_2), \psi_2)dy.$$

Model Transfer via Mean EE in an AM System

Deviation under $x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2$: $\Delta_i(\theta, x_1, x_2)$.

Model under fixed $c_2 \in \mathcal{X}_2$: $\Delta_i(\theta, x_1, c_2) = f(\theta, x_1) + \epsilon_\theta$.

Under mean EE, for every $x_1 \in \mathcal{X}_1$ and $x_2 \in \mathcal{X}_2$ there exists a $T_{2\to1}(x_1, x_2) \in \mathcal{X}_1$ such that

$$\Delta_i(\theta, x_1, x_2) = f(\theta, T_{2\to1}(x_1, x_2)) + \epsilon_\theta.$$

Total equivalent amount (TEA) of F_2 in terms of F_1: $T_{2\to1}(x_1, x_2)$.

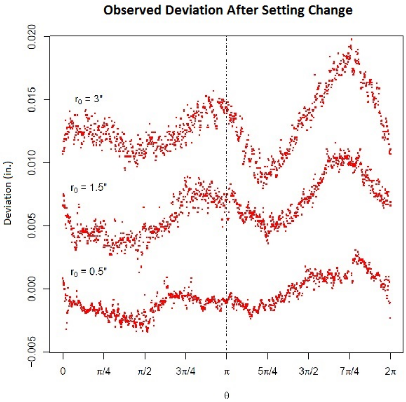
Overview of Bayesian Learning for TEA in Mean EE

A deviation model in a new setting can be obtained by learning the TEA with respect to the mean from the observed data.

Our general Bayesian methodology for learning the TEA with respect to the mean in a new setting proceeds in two steps.

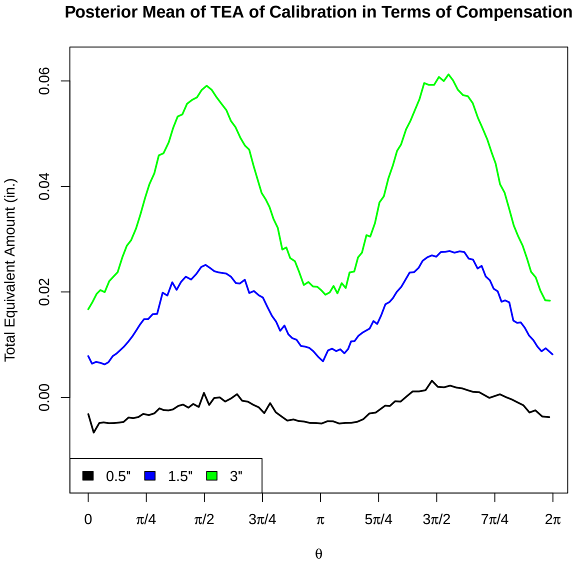
1. Calculate the posterior distribution of the TEA for points under the new setting.
2. Examine the posterior distribution to formulate a model $T_{2\to1}(\mathbf{z}, x_1; \gamma)$ for the TEA in the new setting.

Mean EE of Calibration and Compensation

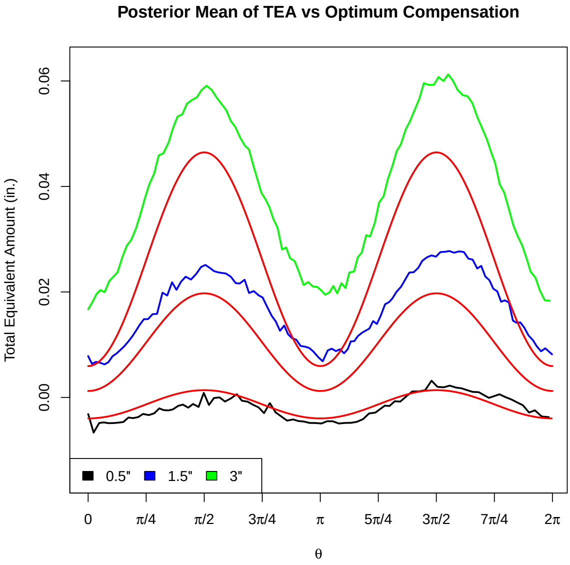


$$\int_{-\infty}^{\infty} y p(y \mid \mathbf{z}, x_1, x_2, \psi) dy = \int_{-\infty}^{\infty} y p_1(y \mid \mathbf{z}, T_{2\to 1}(x_1, x_2), \psi_1) dy$$

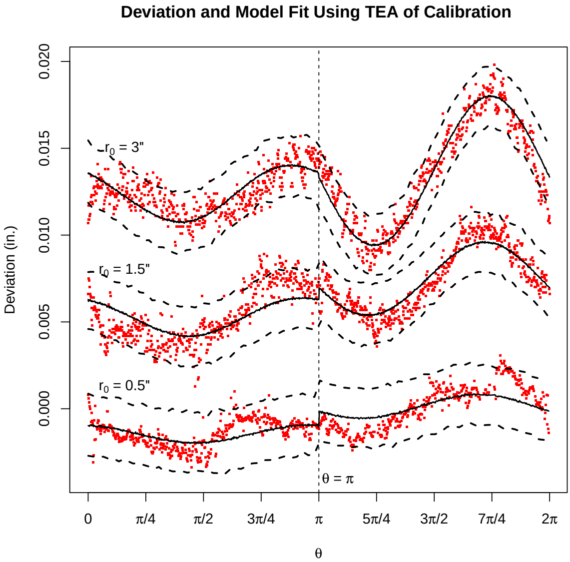
Posterior Distribution of TEA: Calibration



TEA of Calibration vs Optimum Compensation



Calibration TEA Model



Connecting Different Shapes via Deviation Features

The cookie-cutter deviation framework (Huang et al., 2014) connects deviation features in previously manufactured products and new shapes in a modular fashion.

$$\Delta_1(\theta) = \delta_0(\theta \mid \boldsymbol{\alpha}) + \delta_1(\theta \mid \boldsymbol{\beta}) + \epsilon_\theta$$

The δ_0 component captures a global deviation feature shared between two shapes with nominal radius functions $r_0^{\mathrm{nom}}(\cdot)$ and $r_1^{\mathrm{nom}}(\cdot)$.

"Cookie-cutter" δ_1 captures local deformation features unique to the new shape $r_1^{\mathrm{nom}}(\cdot)$.

Bayesian Learning of New Deviation Features

1. Construct the discrepancy measure

$$\Delta_1(\theta) - \delta_0(\theta \mid \tilde{\boldsymbol{\alpha}}),$$

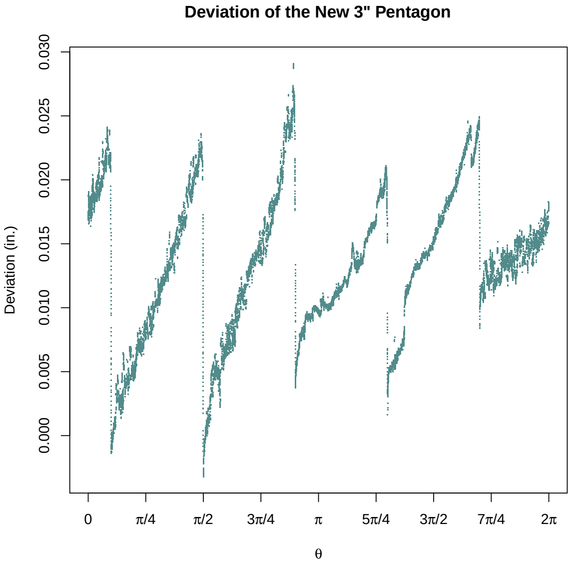
   where $\tilde{\boldsymbol{\alpha}} \sim p(\boldsymbol{\alpha} \mid \mathbf{D}_0)$, to extract information on the local deviation feature δ_1 for a new shape $r_1^{\mathrm{nom}}(\cdot)$.

2. Block and cluster the discrepancy measure distributions according to covariates and discrepancy measure trends that explain the local deviation feature.

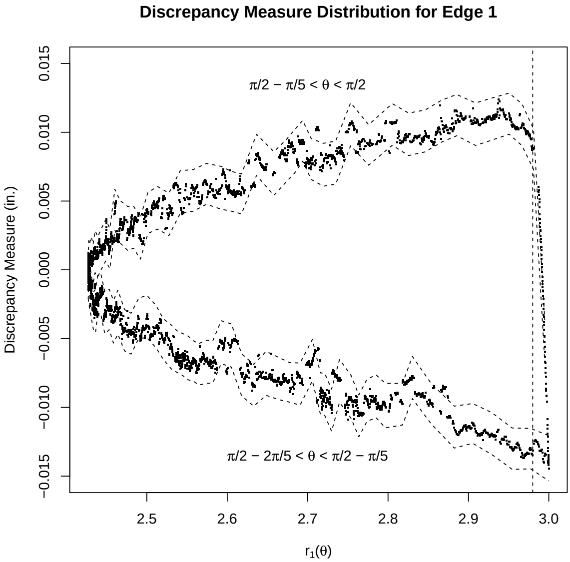
3. Specify a hierarchical model for the parameters $\boldsymbol{\beta}$ of the trends across the blocks.

Learning the Deviation Feature for Straight Edges

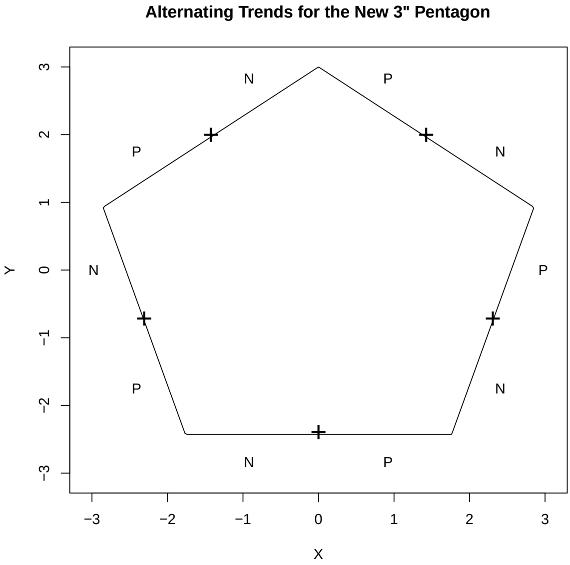
Observed Deviation for 3″ Pentagon



Discrepancy Measure for Edge 1 of the 3″ Pentagon



Clustering Trends for the 3″ Pentagon



Local Deviation Feature Specification for Regular Polygons

$$\delta_1(\theta \mid \boldsymbol{\beta}) = \beta_{0,e(\theta)} + \beta_{1,e(\theta)} s(\theta) \left\{ r_1^{\mathrm{nom}}(\theta) - r_0 \cos\left(\frac{\pi}{n}\right) \right\}^{b_1, e(\theta)} + \beta_{2,e(\theta)} \left\{1 - s(\theta)\right\} \left\{ r_1^{\mathrm{nom}}(\theta) - r_0 \cos\left(\frac{\pi}{n}\right) \right\}^{b_2, e(\theta)},$$

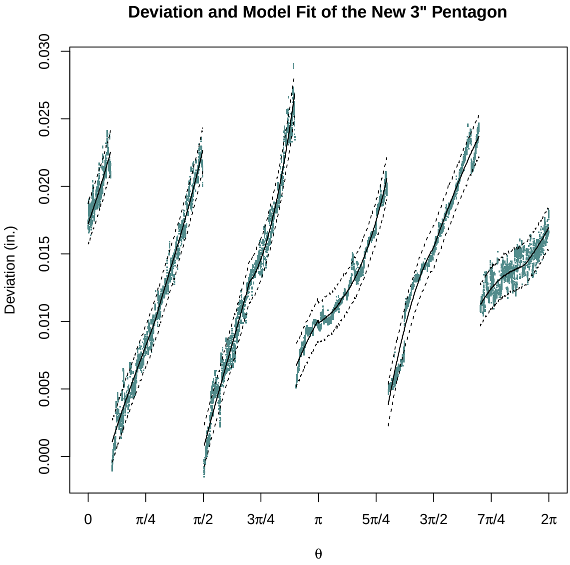
where

$$s(\theta) = \begin{cases} 1 & \text{if } \left(\theta - \frac{\pi}{2}\right) \bmod \frac{2\pi}{n} - \frac{\pi}{n} > 0, \\ 0 & \text{if } \left(\theta - \frac{\pi}{2}\right) \bmod \frac{2\pi}{n} - \frac{\pi}{n} \leq 0, \end{cases}$$

and $e(\theta)$ denotes the edge for θ.

We specify a hierarchical prior on $\boldsymbol{\beta}$, and fit the full model involving both δ_0 and δ_1 simultaneously to the 3$''$ pentagon and the previously manufactured cylinders.

Deviation Model Fit for the Pentagon



Local Deviation Feature Specification for Polygons

$$\delta_1(\theta \mid \boldsymbol{\beta}) = \beta_{0,e(\theta)} + \beta_{1,e(\theta)} s(\theta) \{ r_1^{\mathrm{nom}}(\theta) - r_1^{\mathrm{nom}}(m(\theta)) \}^{b_{1,e(\theta)}} + \beta_{2,e(\theta)} \{1 - s(\theta)\} \{ r_1^{\mathrm{nom}}(\theta) - r_1^{\mathrm{nom}}(m(\theta)) \}^{b_{2,e(\theta)}},$$

where

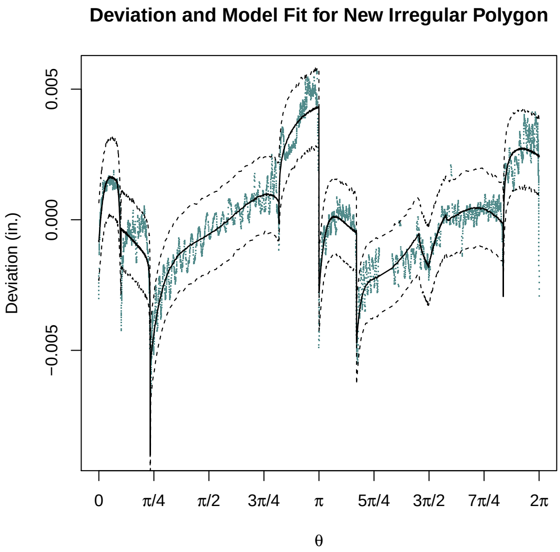
$$m(\theta) = \underset{\substack{t \in [0, 2\pi]: \\ e(t) = e(\theta)}}{\mathrm{argmin}} \; r_1^{\mathrm{nom}}(t),$$

$$s(\theta) = \mathbb{I}\{\theta > m(\theta)\},$$

and $e(\theta)$ denotes the edge for θ.

As before, we specify a hierarchical prior on $\boldsymbol{\beta}$, and fit the full model involving both δ_0 and δ_1 simultaneously to polygons and the previously manufactured cylinders.

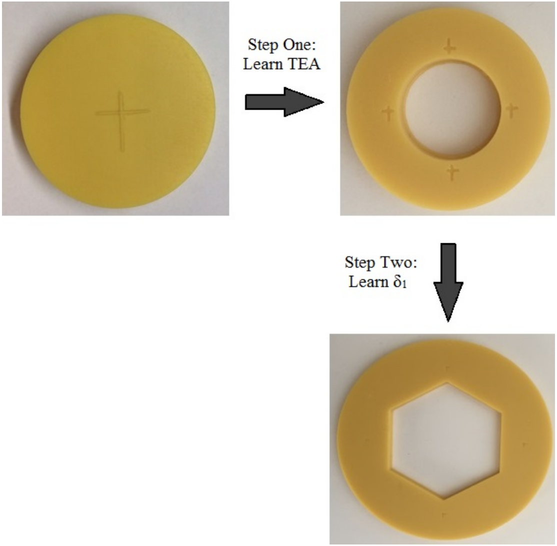
Deviation Model Fit for the Irregular Polygon



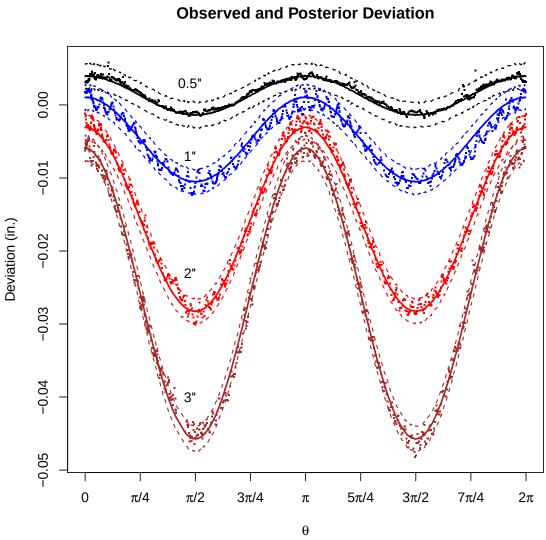
Deviation Modeling for Different Processes and Shapes

1. Infer and model the TEA for a specific shape $r_0^{\mathrm{nom}}(\cdot)$ under a new process condition in terms of compensation.
2. Infer and model the local deviation feature for a new shape $r_1^{\mathrm{nom}}(\cdot)$ manufactured under the new condition.

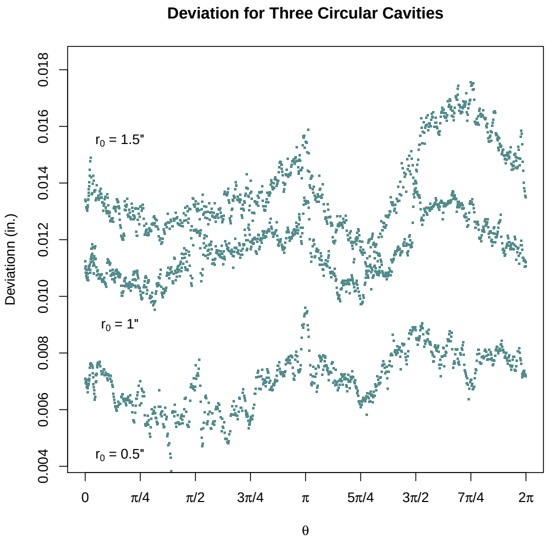
Case Study: Inner Hexagon in Cylinder



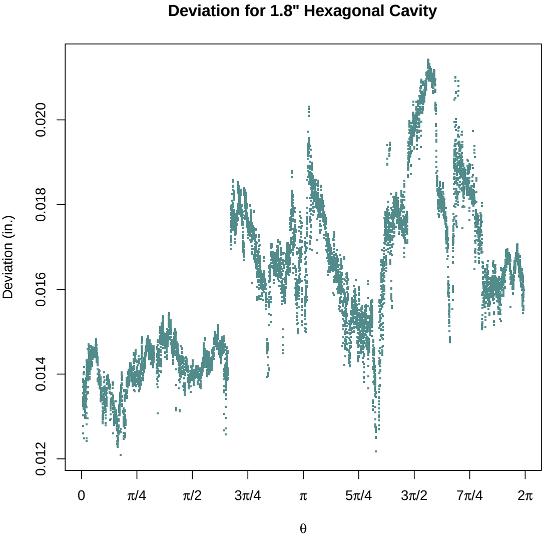
Deviation Data and Model for Cylinders



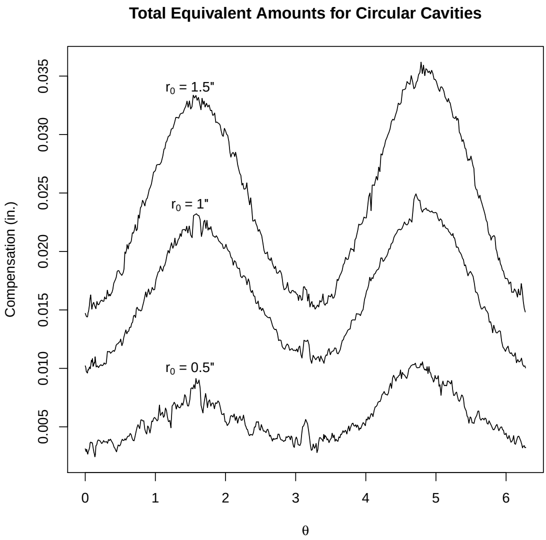
Deviation Data for Circular Cavities



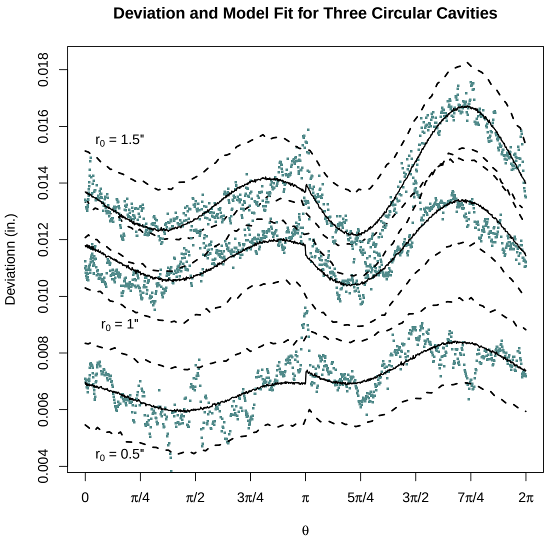
Deviation Data for Hexagonal Cavity



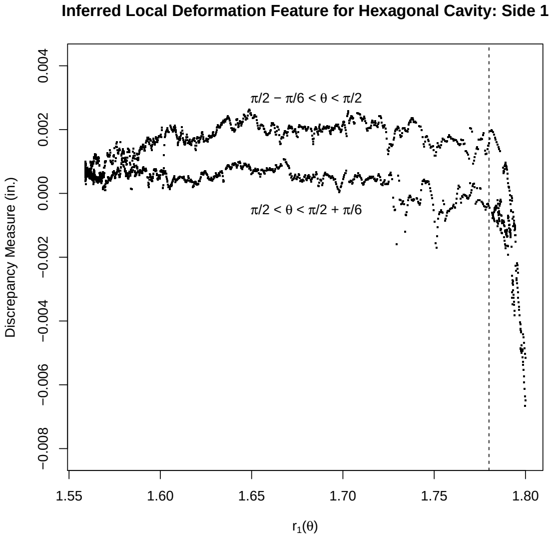
Step One: Learn the TEA for Circular Cavities



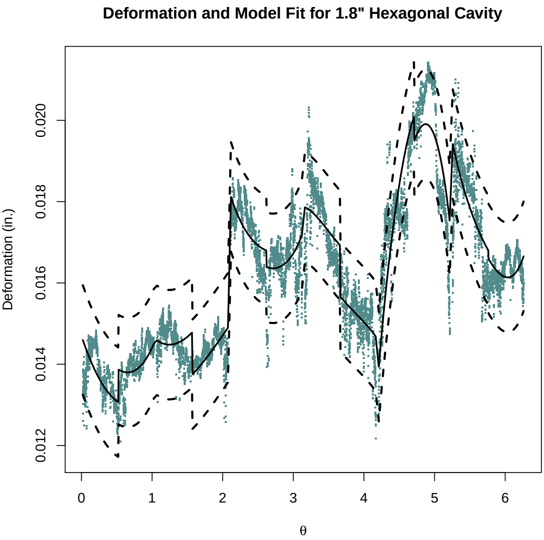
Step One: Deviation Model for Circular Cavities



Step Two: Learn Deviation Feature for Inner Hexagon



Step Two: Deviation Model for Inner Hexagon



Concluding Remarks and Discussion

Our new Bayesian and machine learning methodologies effectively utilize small samples of data to build deviation models for a broad class of disparate shapes across distinct processes in AM systems.

These methodologies are sufficiently general so as to be applied to different types of AM systems.

Next steps:

- Cloud-based app for automated calibration and recalibration of AM systems.
- Prescriptive modeling for different shapes.

Thank You!

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