Selecting an Orthogonal or Nonorthogonal Two-Level Design for Screening

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Two-Level Designs

- One of the possible purposes of a two-level fractional factorial design (FFD) is to screen a large number of factors.
- Initial interest in FFDs focused on regular designs.
- Regular FFDs have orthogonal main effects and possess clear aliasing among factorial effects, but they are restricted to run sizes of 2^k.
- Nonregular FFDs are attractive alternatives, because they exist for any multiple of four.
- While their main effects are also orthogonal, aliasing relations are more complex than regular FFDs.

Nonorthogonal Designs

- The focus of this talk is primarily on orthogonal FFDs, which can be regular or nonregular.
- However, non-orthogonal FFDs for screening purposes have been developed as well.
- Of particular interest are...

Nonorthogonal Designs

- Bayesian *D*-optimal designs (DuMouchel and Jones (1994, *Technometrics*); Jones et al. (2008, *JSPI*))

- MEPI-optimal designs (Smucker and Drew (2015, *Technometrics*)): designs that optimize evaluation of model with all main effects and a selected number of two-factor interactions

- PEC-optimal designs (Smucker et al. (2012, *Technometrics*)): designs that optimize evaluation of models with all main effects and all those two-factor interactions in subsets of the factors

Criteria to Rank Two-Level FFDs

There are so many!

- Strength and resolution
- Generalized resolution
- G- and G_2-aberration
- Generalized alias length pattern (galp)
- Estimation and information capacity
- Projection estimation and projection information capacity
- Model discrimination potential
- Minimal dependent sets
- Power

Let's take these one by one.

Strength

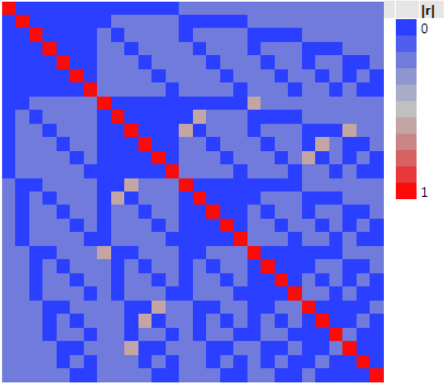
- Strength was introduced in the context of orthogonal arrays
- Let OA(n, k, t) denote an orthogonal array with n rows and k columns and a strength of t; this array projects into an equally replicated full 2^t factorial in every subset of t columns.
- A regular 2^{k-f} fraction with resolution r will have strength $t = r - 1$.
- Strength can be extended to non-orthogonal designs. (All these designs must have a strength smaller than 2. Level-balanced designs have a strength of 1, while designs that are not level-balanced have a strength of 0.)

Generalized Resolution

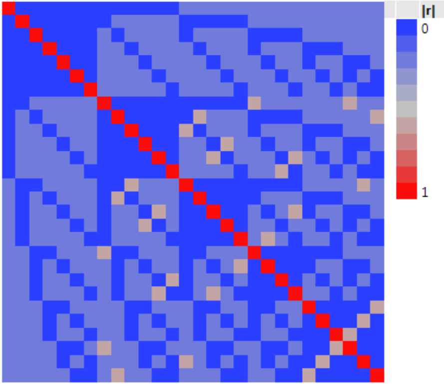
- Consider the two OA$(20, 7, 2)$ designs below. There are 35 three-factor interaction contrasts; for these two designs, every one sums to 4 or -4.
- Implies that the correlation between each main effect and two-factor interaction contrast vector involving three distinct factors is $\pm 4/20$.

Design 20.7.1							Design 20.7.18						
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	1	-1	-1	-1	1	1	1	1
-1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	1	-1
-1	-1	1	1	1	1	-1	-1	-1	1	1	-1	1	1
-1	1	-1	1	-1	-1	1	-1	1	-1	-1	1	-1	1
-1	1	-1	1	1	1	-1	-1	1	-1	1	1	1	-1
-1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	1
-1	1	1	-1	1	-1	-1	-1	1	1	1	-1	-1	-1
-1	1	1	1	1	1	1	-1	1	1	1	1	-1	1
1	-1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1
1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	-1	1
1	-1	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	1
1	-1	1	-1	1	1	1	1	-1	1	-1	1	1	1
1	-1	1	1	1	-1	1	1	-1	1	1	1	-1	-1
1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1	1	1
1	1	-1	-1	1	1	1	1	1	-1	-1	1	1	-1
1	1	-1	1	-1	1	1	1	1	-1	1	-1	-1	1
1	1	1	-1	-1	-1	1	1	1	1	-1	1	-1	-1
1	1	1	1	-1	-1	-1	1	1	1	1	-1	1	-1

Two OA(20, 7, 2)

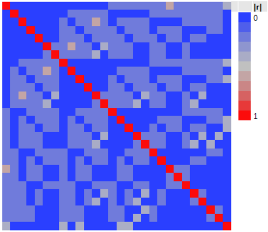


Design 20.7.1

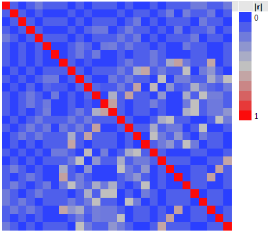


Design 20.7.18

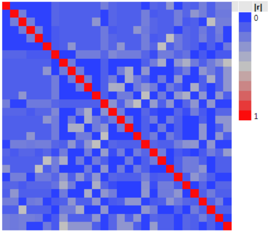
Three Nonorthogonal Designs ($n = 20, k = 7$)



Bayesian D-optimal



MEPI-optimal



PEC-optimal

Generalized Resolution

- Let S denote the maximum sum in absolute value among all the interactions involving $t+1$ columns.
- The generalized resolution ρ is defined as

$$\rho = t + 2 - S/n. \tag{1}$$

- For regular 2^{k-f} fractions, $S = n$ and so $\rho = t + 1$.
- The two $\mathrm{OA}(20, 7, 2)$ designs both have $S = 4$, so their generalized resolution is $\rho = 3.8$.
- For nonorthogonal designs, we have $1 \leq \rho < 3$ and so these will never be preferred over OAs via strength and generalized resolution.

G-Aberration

- The confounding frequency vector (cfv; Deng and Tang (1999)) of a two-level design is the number of interaction columns that sum to each non-zero value.
- For example, Design 20.7.1 has cfv = $[F_3(4) = 35; F_4(12, 4) = (2, 33); F_5(8) = 11; F_6(8) = 1; F_7(4) = 1]$.
- This cfv shows that all 35 three-factor interaction columns sum to ± 4, two four-factor interaction columns sum to ± 12 and the other 33 sum to ± 4, etc.
- Design 20.7.18 has cfv = $[F_3(4) = 35; F_4(12, 4) = (5, 30); F_5(8) = 9; F_6(8) = 1; F_7(4) = 1]$.
- G-aberration is based on sequentially sorting designs according to entries in the cfv.
- So, which of 20.7.1 and 20.7.18 has less G-aberration?

G_2-Aberration

- G_2-aberration (Tang and Deng (1999)) ranks based on the vector $(B_3, B_4, \ldots, B_k)$, called the generalized word length pattern (gwlp).
- Each B_j is the sum of the squares of all the j-factor interaction contrast sums, divided by n^2.
- The table below shows how the gwlp is a compression of the cfv for Design 20.7.1.
- Design 20.7.18's gwlp equals (1.4, 3, 1.44, 0.16, 0.04); it is easy to see that this design has more G_2-aberration than 20.7.1.
- Both G- and G_2-aberration rank orthogonal designs above nonorthogonal designs.

j	cfv	gwlp
3	$F_3(4) = 35$	$B_3 = 35(4/20)^2 = 1.4$
4	$F_4(12, 4) = (2, 33)$	$B_4 = 2(12/20)^2 + 33(4/20)^2 = 2.04$
5	$F_5(8) = 11$	$B_5 = 11(8/20)^2 = 1.76$
6	$F_6(8) = 1$	$B_6 = 1(8/20)^2 = 0.16$
7	$F_7(4) = 1$	$B_7 = 1(4/20)^2 = 0.04$

The Q_B Criterion

- The Q_B criterion (Tsai et al. (2007)) is to minimize the weighted average of an approximation to the A_s-criterion over all sub-models of the two-factor-interaction model.
- Each sub-model is weighted with the prior probability that the model turns out to be the best model.
- Sub-models satisfy the marginality requirement that a main effect may be omitted only if that factor does not appear in any interaction.
- Assume strong heredity prior where the prior probability of a main effect being active is π_1, and the conditional probability that an interaction is active is π_2 if both main effects are active and zero otherwise.
- Under these assumptions, the Q_B criterion is calculated as

$$Q_B = \{[\xi_{10}+2(k-1)\xi_{21}]B_1+[2\xi_{20}+\xi_{21}+2(k-2)\xi_{32}]B_2+6\xi_{31}B_3+6\xi_{42}B_4\}/n,$$

where $\xi_{ij} = \pi_1^i \pi_2^j$.

Q_B Criterion

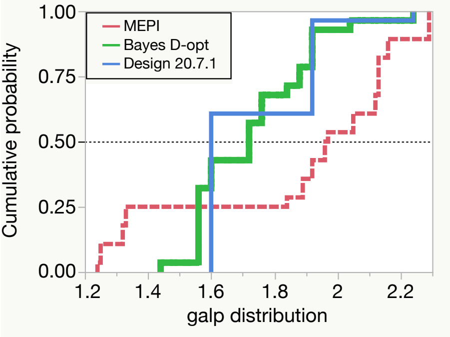
- Suppose (π_1, π_2) = (0.5, 0.8).
- Then, for $k = 7$, $Q_B = (2.9B_1 + 1.5B_2 + 0.6B_3 + 0.24B_4)/n$.
- The following table compares two orthogonal designs and three non-orthogonal designs, ordered by Q_B (smaller is better).
- Observe how this criterion ranks the MEPI and Bayes-D designs higher than the poorer of the two orthogonal designs.

Design	B_1	B_2	B_3	B_4	Q_B
20.7.1	0.00	0.00	1.40	2.04	0.066
20.7.MEPI	0.04	0.16	0.48	3.16	0.070
20.7.Bayes-D	0.00	0.04	1.68	1.64	0.073
20.7.18	0.00	0.00	1.40	3.00	0.078
20.7.PEC	0.10	0.18	1.00	2.00	0.082

Generalized Alias Length Pattern

- The generalized alias length pattern (galp; Cheng et al. (2008)) is a simple measure of aliasing.
- Let X denote the model matrix for the full two-factor interaction model, and compute galp as the main diagonal of the matrix $(X'X/n)^2$.
- The ith element of this diagonal is the sum of squares for the elements in the ith column of $X'X/n$.
- The minimum value for the ith element is 1, which would indicate that the ith column is uncorrelated with every other column.

galp



- The median value is smallest for Design 20.7.1 and largest for the MEPI design.
- The Bayesian D-optimal and minimum *G*-aberration designs have similar galp distributions.
- The MEPI design has lower aliasing for the main effects but higher aliasing for two-factor interactions.

Estimation Capacity and Information Capacity

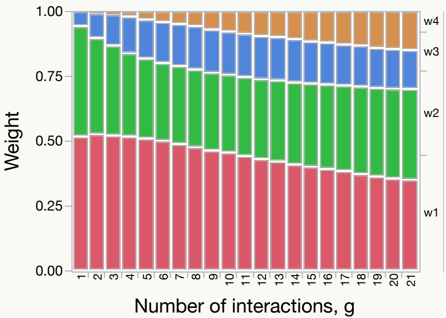
- Estimation capacity (EC) is a vector $(EC_1, EC_2, \ldots, EC_g)$ of the proportions of estimable models with all k main effects and $1, 2, \ldots, g$ two-factor interactions.
- For Design 20.7.1, $(EC_1, EC_2, \ldots, EC_7)$ equals (1, 1, 1, 1, 1, 0.99994, 0.9996).
- All models with 7 main effects and 5 two-factor interactions are estimable; EC_6 is less than 1 because 3 of the 54,264 possible models with 7 main effects and 6 interactions are not estimable.
- Assuming that at most 7 of the 21 possible two-factor interactions are active and no higher-order interactions are present, one is essentially guaranteed that the true model will be estimable.

Information Capacity

Design	# interactions *g*						
	Estimation Capacity (EC)						
20.7.1	1	1	1	1	1	0.9999	0.9996
20.7.18	1	1	1	1	1	0.9997	0.9980
20.7.MEPI	1	1	1	1	1	0.9998	0.9987
20.7.Bayes-D	1	1	1	1	1	1	0.9999
20.7.PEC	1	1	1	1	1	1	1
	Information Capacity (IC)						
20.7.1	0.9755	0.9512	0.9266	0.9011	0.8745	0.8461	0.8153
20.7.18	0.9755	0.9491	0.9207	0.8902	0.8572	0.8212	0.7808
MEPI	0.9670	0.9546	0.9378	0.9170	0.8926	0.8644	0.8318
BayesD	0.9637	0.9345	0.9064	0.8785	0.8503	0.8211	0.7902
PEC	0.9412	0.9207	0.8990	0.8758	0.8511	0.8245	0.7956

- The estimation capacity is excellent for all these designs.
- For nonregular and nonorthogonal designs, models may be estimable but with D-efficiency $< 100\%$, so estimation capacity alone disregards information.
- Designs can be compared by augmenting *EC* with the mean *D*-efficiency across all models of equal size (or information capacity (*IC*)).
- Given the similarity of the *EC* vectors, *IC* is a more useful criterion.

More on Information Capacity



- From Cheng et al. (2002): minimizing average trace$[(X_g'X_g/n)^2]$ across all possible models with g interactions is a good surrogate for maximizing the average determinant of $X_g'X_g/n$.
- Average trace$[(X_g'X_g/n)^2]$ equals

$$1+k+g+2[1+\frac{g}{G}(k-1)]B_1+2[1+\frac{g}{G}+\frac{g(g-1)}{G(G-1)}(k-2)]B_2+6\frac{g}{G}B_3+6\frac{g(g-1)}{G(G-1)}B_4,$$

  where $G=0.5k(k-1)$.
- OAs will often be preferred for screening but not necessarily estimation of the full two-factor interaction model.

Projection Estimation Capacity

- Projection estimation capacity (*PEC*; Loeppky et al. (2007)) is based on the proportion of subsets of the *k* factors for which the two-factor interaction model is estimable.
- When the analysis is based on first identifying which factors are active, *PEC* is well-motivated.
- For Design 20.7.1, the two-factor interaction model can be estimated for any subset of four factors and for 19 out of the 21 five-factor projections; hence $p_4 = 1$ and $p_5 = 19/21$.
- Design 20.7.18 is slightly worse, with $p_5 = 18/21$, while all three non-orthogonal designs have $p_5 = 21/21$.
- The following table shows the PIC sequence for each design.

Design	# factors			
20.7.1	1	0.9827	0.9328	0.7584
20.7.18	1	0.9827	0.9226	0.6737
20.7.MEPI	0.9903	0.9766	0.9201	0.7790
20.7.Bayes-D	0.9990	0.9759	0.9198	0.8111
20.7.PEC	0.9801	0.9507	0.8886	0.7756

Model Discrimination

- Jones et al. (2007, *JSPI*) observed that designs with the same estimation capacity may differ considerably in model discrimination capabilities.
- To identify the correct model, in addition to being estimable it must stand out as a superior fit to the data.
- They propose several criteria to quantify potential for model discrimination.
- Expected Prediction Difference (EPD):

$$EPD = trace[H_1 - H_2]^2/n,$$

  where H_1 and H_2 are the hat matrices corresponding to the two models.
- They proposed computing the average and the minimum EPD over all pairs of models being considered.

Model Discrimination

Design	# interactions				# interactions			
	1	2	3	4	1	2	3	4
	AvgEPD				MinEPD			
20.7.Bayes-D	0.0952	0.1640	0.2187	0.2574	0.0666	0.0293	0.0217	0.0132
20.7.1	0.0951	0.1638	0.2182	0.2563	0.0510	0.0339	0.0265	0.0118
20.7.PEC	0.0947	0.1626	0.2161	0.2538	0.0548	0.0343	0.0245	0.0149
20.7.MEPI	0.0940	0.1599	0.2108	0.2455	0.0494	0.0330	0.0199	0.0088
20.7.18	0.0937	0.1588	0.2079	0.2396	0.0360	0.0238	0.0093	0.0048

- The Bayesian D-optimal design is best in terms of average EPD, but the minimum *G*-aberration design is better with respect to the minimum EPD for models with two or three interactions.
- The MEPI design, which performed well in terms of estimation capacity is near the bottom in terms of discrimination for the main effect plus up to 4 interactions.
- Good estimation capacity does not imply better model discrimination.

Some Take-Aways from the Review of Criteria

1. In terms of computational complexity: strength, gwlp, Q_B, and galp are simplest to compute followed by cfv, PEC/PIC, EC/IC, and model discrimination criteria.
2. Generalized resolution and strength are also applicable to nonorthogonal designs but never preferred through these criteria.
3. Confounding frequency vector and G-aberration exploit orthogonality and are not suitable for nonorthogonal designs.
4. While G_2-aberration can be applied to nonorthogonal designs, it will always prefer orthogonal designs.
5. Q_B is a weighted sum of gwlp and may rank nonorthogonal designs higher than orthogonal designs.
6. In terms of information capacity, the fewer the number of interactions expected, the more justification one has for using an orthogonal array.
7. Differences in PIC tend to be larger than differences in IC. PIC should be preferred for design selection.

Power and False Discovery Rates

- When a final comparison is to be made between several competing designs, it is useful to speculate a set of possible models and employ simulation to compare power to detect active effects.
- A measure of false positive results should be included as well to check whether good power is not mitigated by a high error rate.
- We measure false discovery rates (FDR) as well as power in a simulation study, addressing detection of main effects and two-factor interactions separately.

Selected 7-factor Designs

n	ID	ρ	B_1	B_2	B_3	B_4	Q_B	x: p_x	PIC_x
16	16.7.1	4.0	0	0	0	7	0.105	4: 0.8	0.80
	16.7.4	3.5	0	0	2	3	0.120	4: 1	0.89
	16.7.5	3.5	0	0	2	3.5	0.128	4: 1	0.89
	MEPI		0.047	0.188	0.672	5.125	0.128	4: 0.829	0.76
	BAYESD	3.0	0	0	2.5	2.5	0.131	4: 0.886	0.80
	PEC		0.094	0.219	2.125	2.813	0.159	4: 1	0.84
20	20.7.1	3.8	0	0	1.4	2.04	0.066	5: 0.905	0.76
	MEPI		0.04	0.16	0.48	3.16	0.070	5: 1	0.78
	BAYESD		0	0.04	1.68	1.64	0.073	5: 1	0.81
	20.7.18	3.8	0	0	1.4	3	0.078	5: 0.857	0.67
	PEC		0.1	0.18	1	2	0.082	5: 1	0.78
24	BAYESD	3.67	0	0	0.667	1.667	0.033	5: 1	0.90
	MEPI		0	0.028	0.472	2.167	0.035	5: 1	0.89
	24.7.1	4.67	0	0	0	3.889	0.039	5: 1	0.87
	PEC		0.028	0.083	0.806	1.333	0.042	5: 1	0.88
28	MEPI		0	0.143	0.163	1.122	0.021	6: 1	0.86
	28.7.1	3.86	0	0	0.714	0.878	0.023	6: 1	0.87
	BAYESD		0.02	0.082	0.571	0.796	0.026	6: 1	0.86
	PEC		0.026	0.112	0.566	0.735	0.027	6: 1	0.86
32	32.7.2	4.5	0	0	0	1.5	0.011	6: 0.857	0.78
	D-Opt		0.043	0.109	0.293	0.406	0.018	6: 1	0.92
	32.7.x	3.75	0	0	0.812	0.375	0.018	6: 1	0.90

Selected 11-factor Designs

n	ID	ρ	B_1	B_2	B_3	B_4	Q_B	x: p_x	PIC_x
20	MEPI		0.08	0.4	1.8	38.8	0.173	5: 1	0.76
	20.11.1	3.4	0	0	8.2	22.8	0.191	5: 0.848	0.67
	BAYESD		0	0	9.48	18.96	0.199	5: 0.719	0.57
	PEC		0.06	0.66	9.12	17.68	0.229	5: 0.887	0.63
24	24.11.1	4.67	0	0	0	36.67	0.092	5: 1	0.87
	MEPI	4.67	0	0	0	36.67	0.092	5: 1	0.87
	BAYESD		0.097	0.208	6.83	13.94	0.139	5: 0.998	0.80
	PEC		0.125	0.847	6.028	15.33	0.161	5: 1	0.76
32	32.11.1	4.0	0	0	0	25.5	0.048	6: 0.221	0.18
	MEPI		0	0.25	0	24.5	0.053	6: 0.264	0.21
	32.11.x	3.75	0	0	4.25	15.75	0.069	6: 0.983	0.76
	BAYESD		0.07	0.18	4.469	9.16	0.070	6: 1	0.79
	PEC		0.035	0.305	3.957	11.38	0.070	6: 1	0.79
40	40.11.1a	4.4	0	0	0	18.96	0.028	6: 0.831	0.70
	40.11.1b	4.4	0	0	0	18.96	0.028	6: 0.799	0.67
	40.11.1c	4.4	0	0	0	18.96	0.028	6: 0.701	0.60
	MEPI		0	0.24	0	18.16	0.033	6: 0.851	0.69
	40.11.260	4.4	0	0	0	22.8	0.034	6: 0.468	0.39
	PEC		0.03	0.210	2.7	8.18	0.039	6: 1	0.86
	BAYESD		0.053	0.295	2.953	5.8	0.041	6: 1	0.86
48	48.11.1	4.67	0	0	0	9.11	0.011	6: 0.935	0.88
	48.11.x	4.67	0	0	0	10.89	0.014	6: 0.974	0.90
	MEPI		0.009	0.073	0.745	8.54	0.017	6: 1	0.92
	BAYESD		0.059	0.156	2	4.14	0.024	6: 1	0.91
	PEC		0.045	0.149	2.160	6.12	0.026	6: 1	0.89

Simulation Protocol

For each design under comparison, our simulation is carried out as follows. In each of 1,000 iterations:

1. From the columns of the design matrix, m columns are randomly assigned as the active main effects.
2. Based on the selection of active main effects, g two-factor interaction columns are randomly assigned as active under the assumption of weak effect heredity.
3. The coefficients, β, for the active effects are obtained via two scenarios:
   - Main effects and two-factor interactions are of the same size: randomly sample from $\{0.5, 1, 1.5, 2, 2.5, 3, 3.5\}$. A sign (+ or -) is randomly applied to each coefficient.
   - Two-factor interactions are smaller than main effects.
4. Response vector is generated as $y = X\beta + \epsilon$ with $\epsilon_i \sim N(0, 1)$.
5. The set of active effects is decided by performing one of the analysis methods.

At the end of 1,000 iterations, we compute power and FDR overall and separately for main effects and two-factor interactions.

Analysis Method: Forward Selection

- Despite documented shortcomings (e.g., high Type I error rates) forward selection remains popular and commonly used in practice for variable selection.
- Begins with the null model and adds the most significant term at each step based on an F-test.
- We perform forward selection restricted by weak effect heredity (an interaction term is not eligible to enter the model unless at least one of its parent main effects is selected for inclusion).
- To help avoid model overfitting, we control the experiment-wise error rate (EER) via Bonferroni adjusted p-values.
- Terminates when the adjusted p-value first exceeds the specified EER. We use EER=0.5.

Analysis Method: Dantzig Selector

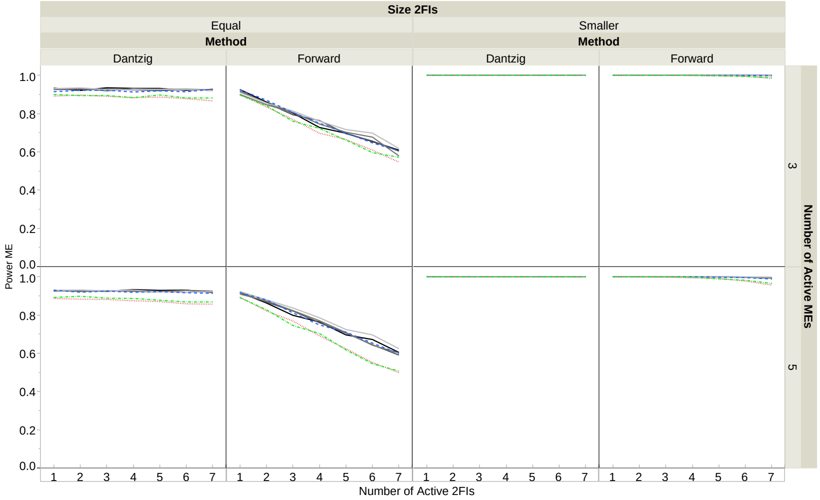
- The Dantzig selector is a shrinkage method in which the estimator $\hat{\beta}$ is the solution to

$$\min_{\hat{\beta}\in\mathbb{R}} \left\|\hat{\beta}\right\|_1 \text{ subject to } \left\|\mathbf{X}'(\mathbf{y}-\mathbf{X}\hat{\beta})\right\|_\infty \leq \delta,$$

  where δ is a tuning constant.
- Choose the value of the tuning parameter δ via the modified AIC (AIC_c): $\text{AIC}_c = n\log\left(\frac{RSS}{n}\right) + \frac{2n\tilde{p}}{n-\tilde{p}-1}$, where RSS is the residual sum of squares and $\tilde{p}$ is the number of terms in the model under consideration.

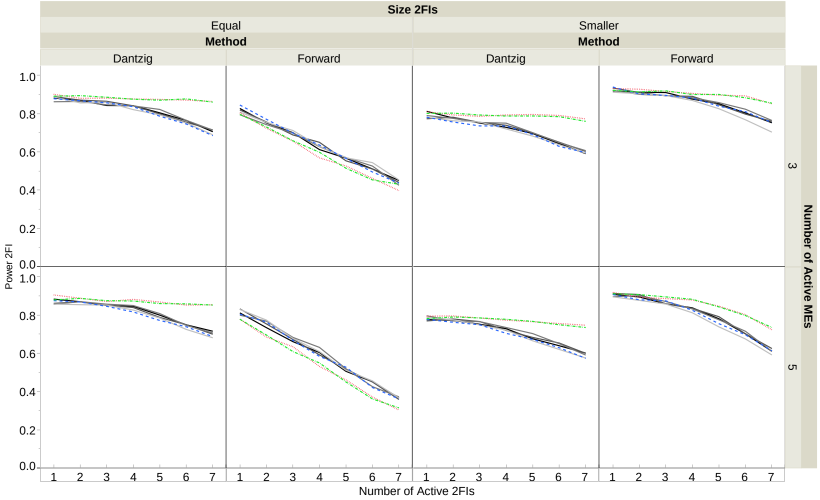
Simulation Results ($k = 11, n = 40$)

Power for Main Effects



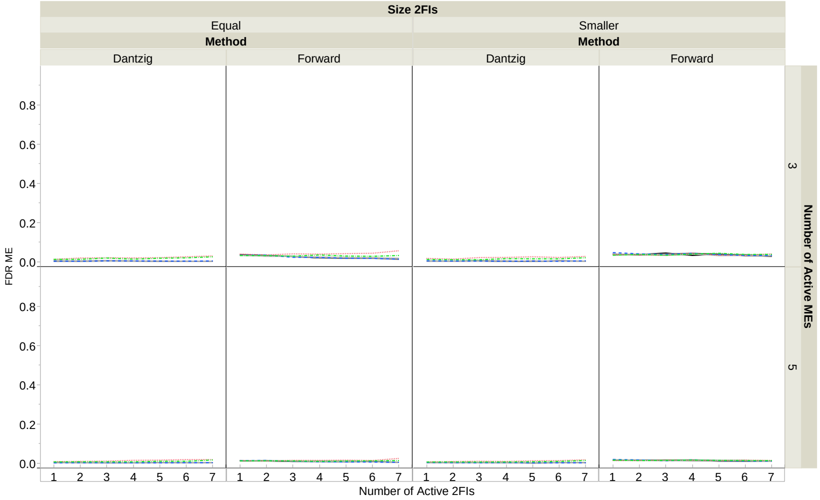
Simulation Results ($k = 11, n = 40$)

Power for Two-Factor Interactions



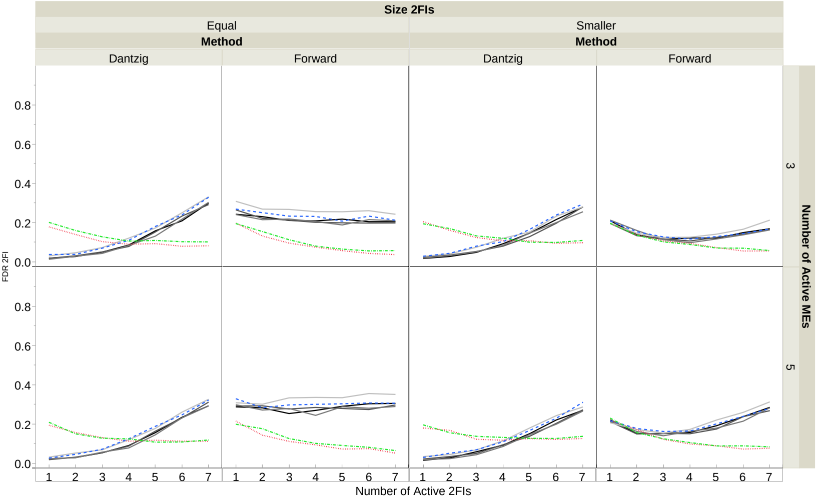
Simulation Results ($k = 11, n = 40$)

FDR for Main Effects



Simulation Results ($k = 11, n = 40$)

FDR for Two-Factor Interactions

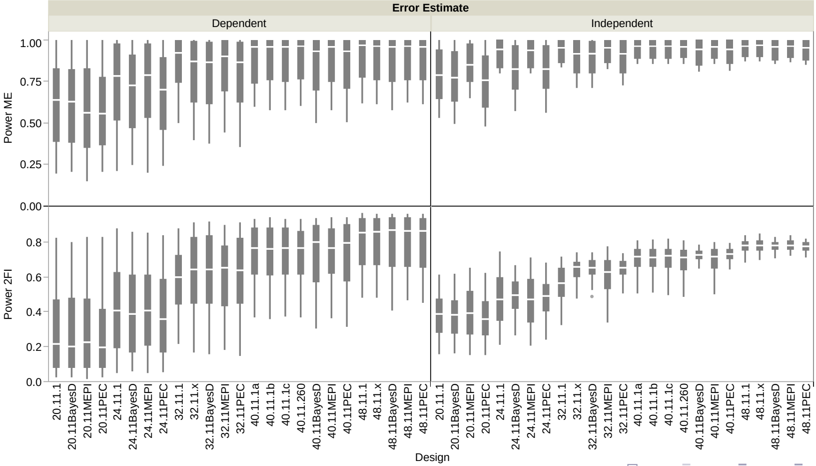


Take-Aways from the Simulation Study

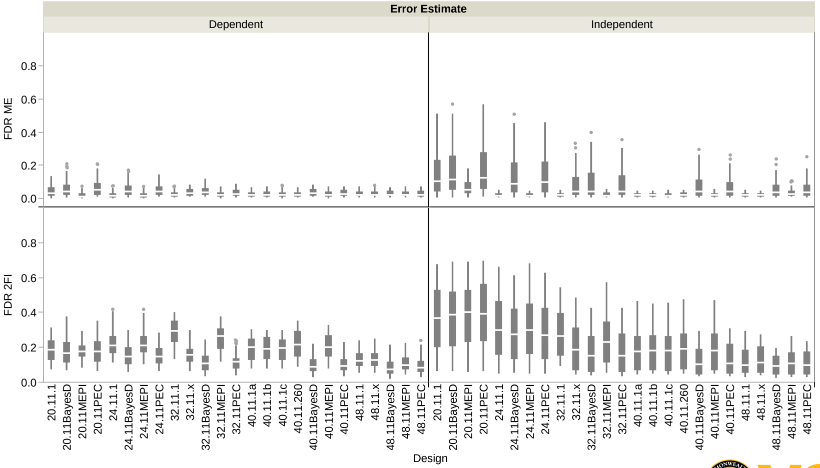
- Dantzig Selector outperforms forward selection except for detection of 2FIs when smaller than main effects
- OAs and MEPI-optimal designs are powerful for detecting main effects (control error rate better than other designs, too)
- Bayesian *D* and PEC-optimal designs stand out for interaction detection
- Few differences observed among OAs

Partial Replication - Power

Suppose we have an independent error estimate available (based on 3df) and this is used in the forward selection procedure instead of the usual model dependent mean square error.



Partial Replication - FDR



Practical Experiment

- Researchers were involved in making phantoms to calibrate medical devices.
- Phantoms are cylindrical pieces of gelatinous material that mimic human tissues.
- A phantom is tested by exposing it to light of various wavelengths. For each wavelength, reflection is recorded, which can be affected by the absence or presence of seven ingredients.
- Experiment used was the OA(40,7,3).
- Models indicated 3 substantial main effects and two interactions (smaller than main effects).
- Might we have been successful with a smaller experiment?

Alternatives for Phantom Experiment (3 MEs and 2 2FIs)

		Forward		Dantzig	
n	Design	ME	2FI	ME	2FI
16	16.7.1	0.99 / 0.03	0.36 / 0.49	1.00 / 0.04	0.27 / 0.66
16	16.7.4	0.95 / 0.04	0.62 / 0.24	0.98 / 0.05	0.57 / 0.38
16	16.7.5	0.96 / 0.04	0.62 / 0.21	0.98 / 0.05	0.57 / 0.39
16	MEPI	0.97 / 0.03	0.59 / 0.27	1.00 / 0.03	0.55 / 0.32
16	Bayes-D	0.96 / 0.06	0.55 / 0.21	0.98 / 0.08	0.52 / 0.31
16	PEC	0.93 / 0.03	0.59 / 0.20	0.98 / 0.03	0.56 / 0.30
20	20.7.1	1.00 / 0.03	0.81 / 0.18	1.00 / 0.03	0.77 / 0.21
20	20.7.18	0.99 / 0.04	0.79 / 0.17	1.00 / 0.03	0.75 / 0.22
20	MEPI	1.00 / 0.04	0.79 / 0.18	1.00 / 0.02	0.71 / 0.17
20	Bayes-D	0.99 / 0.03	0.80 / 0.16	1.00 / 0.03	0.77 / 0.20
20	PEC	0.99 / 0.04	0.80 / 0.16	1.00 / 0.03	0.72 / 0.20
24	24.7.1	1.00 / 0.04	0.85 / 0.15	1.00 / 0.01	0.80 / 0.11
24	MEPI	1.00 / 0.04	0.87 / 0.16	1.00 / 0.01	0.78 / 0.16
24	Bayes-D	1.00 / 0.03	0.85 / 0.16	1.00 / 0.03	0.82 / 0.17
24	PEC	1.00 / 0.03	0.85 / 0.15	1.00 / 0.03	0.80 / 0.15
28	28.7.1	1.00 / 0.03	0.91 / 0.14	1.00 / 0.03	0.86 / 0.18
28	MEPI	1.00 / 0.05	0.90 / 0.15	1.00 / 0.01	0.86 / 0.16
28	Bayes-D	1.00 / 0.03	0.89 / 0.14	1.00 / 0.03	0.85 / 0.17
28	PEC	1.00 / 0.03	0.89 / 0.15	1.00 / 0.02	0.82 / 0.10
32	32.7.2	1.00 / 0.04	0.91 / 0.15	1.00 / 0.00	0.85 / 0.09
32	32.7x	1.00 / 0.04	0.92 / 0.14	1.00 / 0.02	0.87 / 0.11
32	D-optimal	1.00 / 0.04	0.93 / 0.15	1.00 / 0.01	0.88 / 0.09

- 16-run designs not recommended
- Any of the 20-run designs would be suitable.

Thank you!

Got comments or questions?
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