Planning Fatigue Tests for Polymer Composites

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The 2017 Fall Technical Conference
October 5, 2017

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Outline

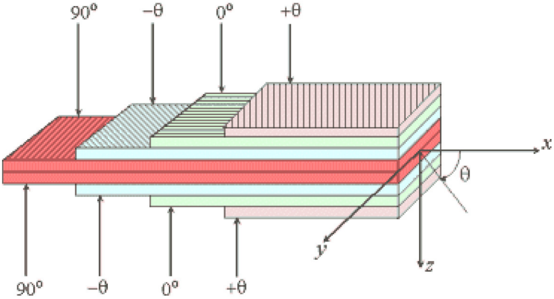
- Background
- Fatigue testing, data, and model
- Statistical optimum test plan
- Comparisons of test plans
- Test plan assessments
- Conclusion and areas for future research

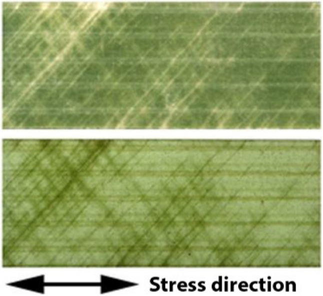
Background

- Polymer composites are widely used in many applications.
- A composite is any material made of more than one component.
- Polymer composites are made from polymers or from polymers along with other kinds of materials.
- Advantages: light, resist heat and corrosion, cost effective.
- Wind turbine blades are usually made of polymer composite because they are lighter than metals thus can provide more energy.

Polymer Composites

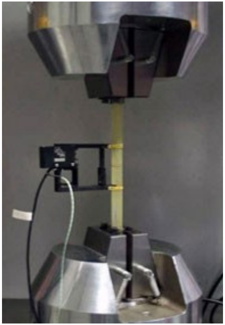
- Polymer composites consist of fibers embedded in a resilient plastic matrix.
- The fiber provides the strength, or reinforcement, for the composite material, and the matrix provides the support.

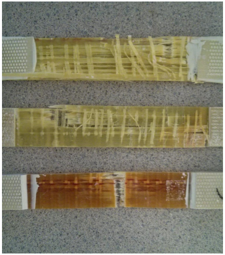




Fatigue Testing

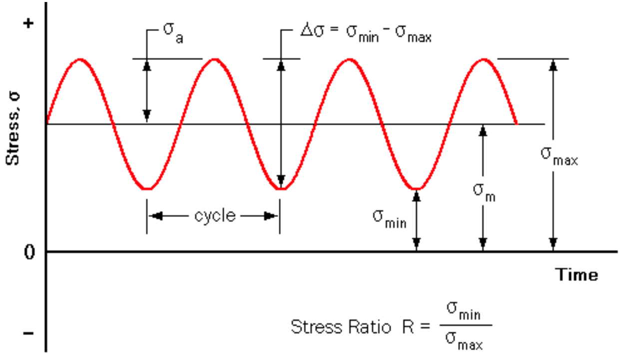
- The fatigue and other properties of the materials need to be tested and to meet certain standards.
- Fatigue occurs when the material is subject to varying levels of stress over a period of time.
- The most common form of fatigue testing is cyclic constant amplitude fatigue testing.
- The current standards: ASTM E739, ASTM D3479





Cyclic Fatigue Testing

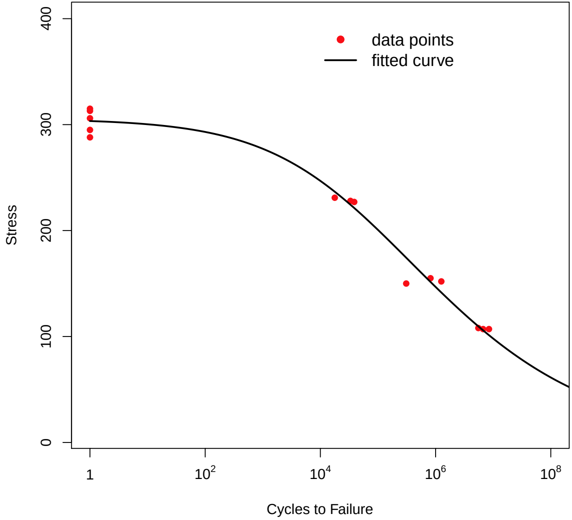
- Cyclic constant amplitude fatigue test is widely used to test the coupon until it fails.
- The number of cycles are recorded.
- Samples are tested under several different level of maximum stress σ_{max}.



The Problem and Objective

- One main goal of fatigue test is to demonstrate that a p proportion of the materials can last a certain number of cycles under the use stress level with some confidence.
- This is related to the estimation of the quantile of the cycles to failure distribution.
- The current standards and engineering practice use balanced designs (i.e., equal allocation of samples).
- The objective of this talk is to applied statistical optimum test planning techniques to polymer composites fatigue testing.
- It is possible to have more accurate estimates, less test duration, and less number of samples to be tested.

The Motivating Dataset



- Five samples tested for ultimate tensile strength σ_U.
- Three stress levels used.
- Three samples for each level.
- The fitted line is the S-N curve.
- Data were re-scaled.

Notation

- The ultimate tensile strength is denoted by σ_u.
- Let s be the number of stress levels and k_i be the samples allocated to stress levels i, $i = 1, \cdots, s$.
- The total sample size is $k = \sum_{i=1}^{s} k_i$.
- σ_i the maximum stress for level i.
- The data are denoted by $\{N_{ij}, d_{ij}\}, j = 1, \ldots, k_i, i = 1, \ldots, s$, where d_{ij} is the censoring indicator.

Model for Cycles to Failure

- The log location scale family of distribution is used to describe the cycles to failure N_{ij} (e.g., lognormal and the Weibull).
- The location parameter is $\mu_i(A, B)$ and scale parameter is ν.
- The model for $\mu_i(A, B)$ is based on Epaarachchi and Clausen (2003):

$$\mu_i(A, B) = \frac{1}{B} \log \left\{ \left(\frac{B}{A} \right) f^B \left(\frac{\sigma_u}{\sigma_i} - 1 \right) \left(\frac{\sigma_u}{\sigma_i} \right)^{\gamma(\alpha)-1} [1 - \psi(R)]^{-\gamma(\alpha)} + 1 \right\}$$

- The unknown parameters are $\boldsymbol{\theta} = (A, B, \nu)'$.

Parameter Interpretations

- Parameter A represents environmental effects on the fatigue process.
- Parameter B is material specific.
- The parameter ψ is a function of the ratio $R = \sigma_{\text{min}}/\sigma_{\text{max}}$, where

$$\psi(R) = \begin{cases} R & -\infty < R < 1 \\ \frac{1}{R} & 1 < R < \infty \end{cases}.$$

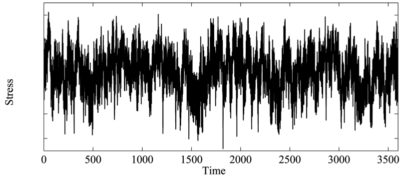
- The parameter $\gamma(\alpha) = 1.6 - \psi|\sin(\alpha)|$ is a function of the smallest angle α between the testing direction and the fiber direction.
- Parameter f is the frequency of the cyclic testing procedure.

Quantile of Cycles to Failure

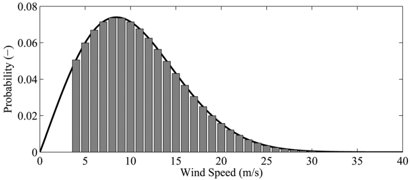
- The p quantile of the cycles to failure distributions at a use condition σ_{use} is denoted by $N_p(\sigma_{\text{use}})$.
- The parameter $\boldsymbol{\theta}$ is estimated by using the maximum likelihood (ML).
- Let $\widehat{N}_p(\sigma_{\text{use}})$ be the estimator, obtained by substituting in $\widehat{\boldsymbol{\theta}}$.
- The large sample variance of is $\text{AVar}\left\{\log\left[\widehat{N}_p(\sigma_{\text{use}})\right]\right\}$, which is calculated by using the Fisher information matrix and the delta method (formulae available in paper).

Multiple Use Conditions

- Most existing work focuses on test planning under a single use condition.
- In real application, the stress under the use condition is time varying.
- While it is challenging to consider test planning under random use profile, we consider test planning under multiple use conditions.



Wind Turbine Blade Stress



Wind Speed Distribution

Test Plan Setup

- We consider there is a range of use conditions, $\sigma_{\text{use},l}, l = 1, \cdots, L$, and we know their relative frequency, $\xi_l, l = 1, \cdots, L$.
- Let N_m be the maximum number of cycles allowed for a test at a single stress level.
- Let $q_i = \sigma_i / \sigma_u$ represent the maximum stress at level i subject to $q_i \in [q_L, q_U]$, where q_L and q_U are the pre-specified lower and upper bounds of the planning range.
- Let $\pi_i = k_i / k$ be the proportion of the total sample size allocated to level i.
- Let $\boldsymbol{\eta} = (q_1, \cdots, q_s, \pi_1, \cdots, \pi_s)'$ be the vector of design parameters.

Statistical Optimum Design

- The statistical optimum design is obtained by minimizing:

$$\min_{\boldsymbol{\eta}} \sum_{l} \xi_l \mathrm{AVar}\left\{\log\left[\widehat{N}_p(\sigma_{\mathrm{use},l})\right]\right\}$$

$$\text{subject to} \quad q_i \in [q_L, q_U], i = 1, \ldots, s,$$
$$\pi_i \in [\pi_{\min}, 1], i = 1, \ldots, s,$$
$$\text{and} \sum_i \pi_i = 1.$$

- Numerical methods are needed to do the optimization.
- A statistical optimum design always ends up with two level of stress for this test planning problem.

Traditional and Compromise Plans

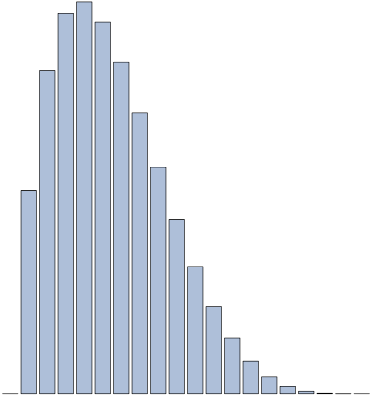
- Traditional Plan
  - Currently used in standards.
  - Use equal allocation.
  - For the motivating example, consists of three stress levels ($q_1 = 0.35, q_2 = 0.50, q_3 = 0.75$) and equal allocation to each level ($\pi_i = 1/3, i = 1, 2, 3$).
- Optimum Plan
  - No constraints on number of stress levels required or amount allocated to each level.
  - Achieve maximum efficiency but may not be robust.
- Compromise Plan
  - Subject to a constraint on the minimum number of stress levels and minimum allocation to each level.
  - With both efficiency and robustness.

Planning Values

- The range for stress q_i is $[0.35, 0.75]$.
- For compromise plans, a minimum distance of 0.10 is enforced for any two stress levels, $\pi_{\text{min}} = 0.10$.
- Take $N_m = 2M$ cycles, the expected fraction failing are $p_L = 0.002$ and $p_U = 1$, for lognormal distribution.
- Take $N_m = 5M$ cycles, the expected fraction failing are $p_L = 0.258$ and $p_U = 1$, for lognormal distribution.
- From the pilot data, $A = 0.00499$, $B = 0.397$, $\nu = 0.421$, $\sigma_u = 303.4$, $f = 3$, and $R = 0.1$.

Use Stress Pattern

- Minimizing the asymptotic variance of the 0.05-quantile of the cycles to failure distribution.
- The use stresses range is from $q_{useL} = 0.05$ to $q_{useL} = 0.25$, with the following relative frequency.



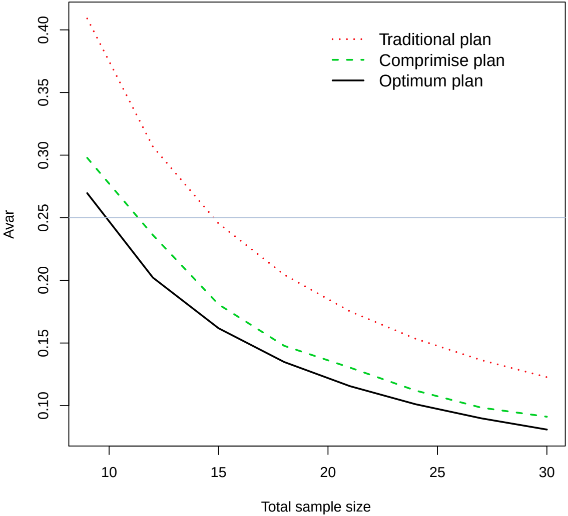
Comparison of Test Plans

- Total sample size $k = 12$.

N_m	AVar	% AVar	Stress Level			Allocation		
			q_1	q_2	q_3	k_1	k_2	k_3
	0.759		0.35	0.50	0.75	4	4	4
$2M$	0.326	57%	0.43	-	0.75	8	-	4
	0.482	36%	0.36	0.46	0.75	3	5	4
	0.306		0.35	0.50	0.75	4	4	4
$5M$	0.202	34%	0.36	-	0.75	8	-	4
	0.236	22%	0.36	0.64	0.75	7	2	3

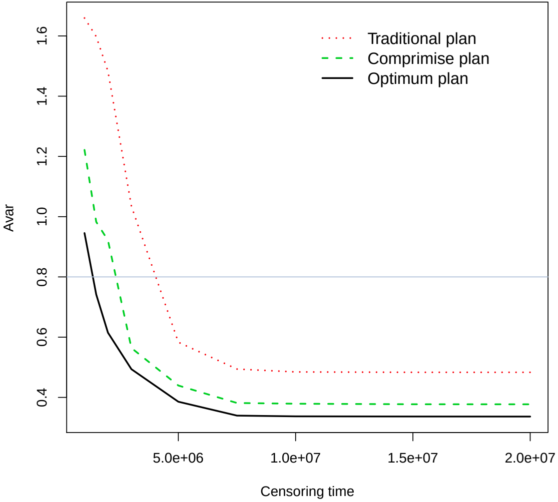
Effect of Sample Size

- Assess the effect of total sample size, $N_m = 5M$.

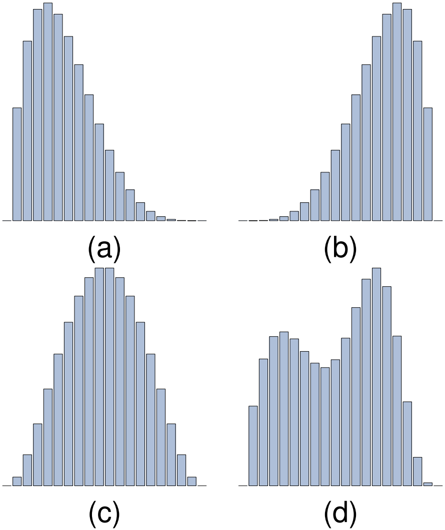


Effect of Censoring Time

- Assess the effect of censoring time, $k = 12$.



Four Different Stress Patterns Under Use Conditions

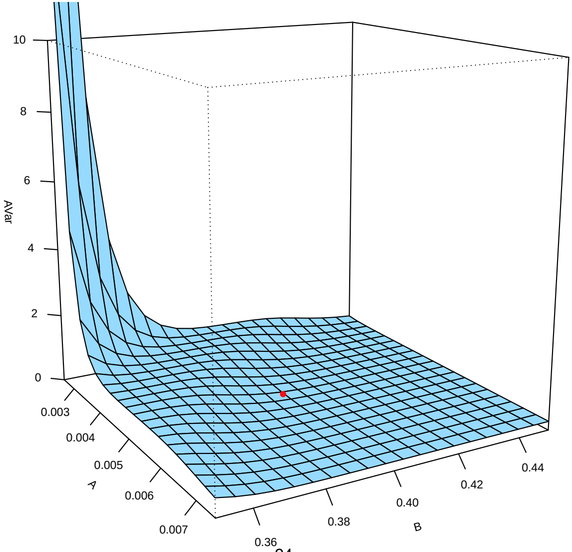


Effect of Use Stress Pattern

- Total sample size $k = 12$ and $N_m = 5M$.

Plan	Pattern	AVar	Stress Level q_1	Stress Level q_2	Stress Level q_3	Allocation k_1	Allocation k_2	Allocation k_3
Trad.	1	0.307	0.35	0.5	0.75	4	4	4
	2	0.155	0.35	0.5	0.75	4	4	4
	3	0.217	0.35	0.5	0.75	4	4	4
	4	0.235	0.35	0.5	0.75	4	4	4
Optimum	1	0.202	0.36		0.75	8		4
	2	0.100	0.35	-	0.75	9	-	3
	3	0.143	0.36	-	0.75	8	-	4
	4	0.155	0.36	-	0.75	8	-	4
Comp.	1	0.236	0.36	0.64	0.74	7	2	3
	2	0.114	0.35	0.65	0.75	8	2	2
	3	0.163	0.36	0.65	0.75	8	2	2
	4	0.181	0.36	0.63	0.75	7	2	3

- Effect of the model parameters, $k = 12$, and $N_m = 5M$.



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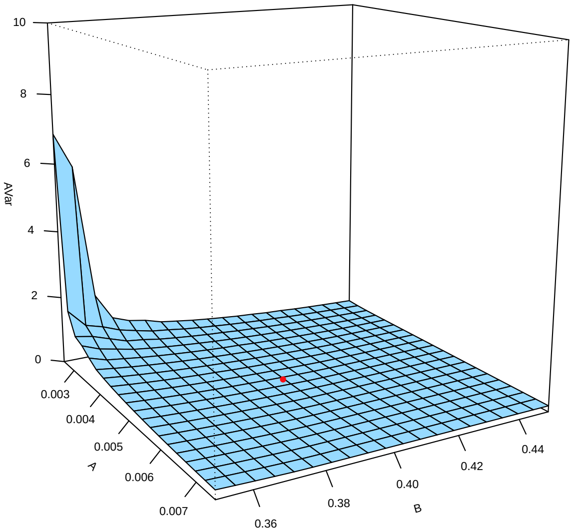
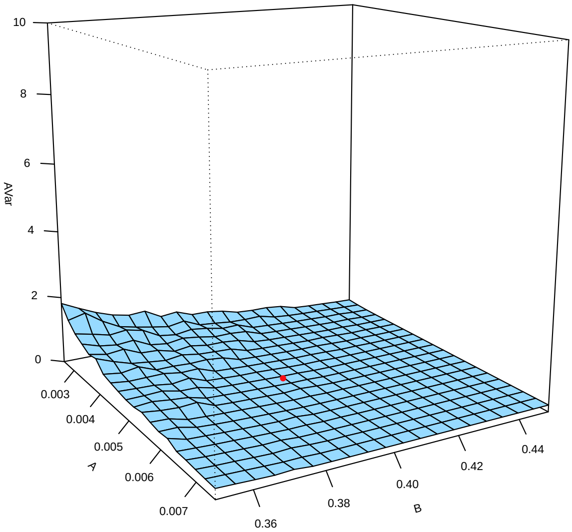


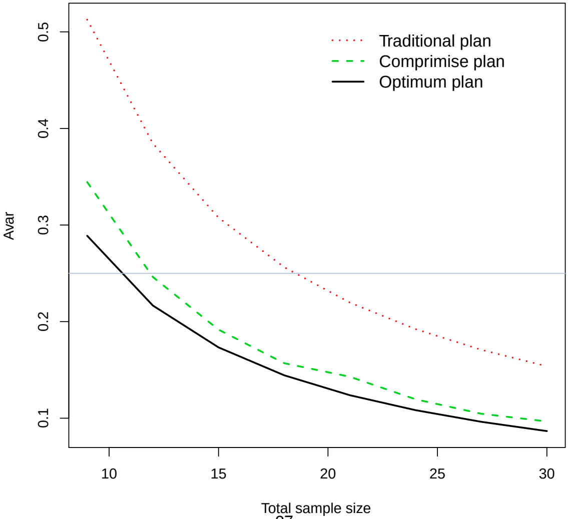
Sensitivity Analysis of Parameter Values–Compromise

- Effect of the model parameters, $k = 12$, and $N_m = 5M$.



Sensitivity Analysis of Distributional Assumption

- Effect of total sample size when the underlying distribution is the Weibull, $N_m = 5M$.



Conclusions

- We applied statistical optimum test planning techniques to fatigue test on polymer composites.
- Both sample size and censoring time will effect on the optimum criterion and test plan configuration.
- We also did sensitivity analysis on the effects parameter values and distributional assumptions.
- Both optimum and compromise plans are more efficient than the traditional plan. The compromise plan is recommended for both the consideration of efficiency and robustness.

Areas for Future Research

- Perform simulation studies to compare exact and large-sample variances at selected design parameters.
- Fatigue testing planning under time varying use profile.
- Fatigue testing planning involves multiple variables (e.g., R values).
- Test planning when the focus is on parameter estimation (e.g., estimate the S-N curve).
- Test planning under block and spectral loading profiles.

Mud and Straw Brick

Thank You with the oldest composite material!

Composite materials have actually been around for quite a long time. As early as 3000 B.C., the ancient Egyptians embedded straw in their mud bricks.